

COT 4210 Homework #10: Sections 7.4 - 7.5

Assigned: 4/6/2021

Due Date/Time: On Webcourses

- 1) Let $\text{MAX-CLIQUE} = \{ \langle G, k \rangle \mid G \text{ is a graph and its largest clique is of size } k \}$. If CLIQUE is in P , prove that MAX-CLIQUE is ALSO in P . (Namely, given a black box that solves the CLIQUE decision problem in polynomial time, design a solution to MAX-CLIQUE in polynomial time.)
- 2) Why are 2 x 3 windows necessary in the proof of the Cook-Levin theorem?
- 3) Show a polynomial time reduction from 4-SAT to 3-SAT, where 4-SAT represents satisfiability for boolean formulas with four literals in each clause instead of 3, and the formula is still in conjunctive normal form. (Namely, show how to transform a boolean formula in 4-SAT form into an equivalent formula in 3-SAT form such that the input formula is satisfiable if and only if the output formula is.)
- 4) The independent set problem is as follows: Given a graph G and an integer k , determine whether or not there are k vertices in G such that no two vertices out of the k share the same edge. Prove that the set INDEPENDENT-SET is NP-Complete via a reduction from a known NP-Complete problem.
- 5) n people live in a house and wish to share their expenses equally. Their respective expenses before settling are x_1, x_2, \dots, x_n . Assume that all of these are greater than 0. They agree to write each other checks so as to make each person's expenses equal the average cost. Naturally, they want to minimize the number of checks written. Formalize this as a decision problem and prove that it is NP-Complete.