

### Weekly Proof Questions (Sections 3.1 – 3.3)

Assigned: 2/24/2015

Due: 2/26/2015

1) Give the sequence of configurations that the Turing machine  $M_2$  in chapter 3 of the text (and shown in class) goes through while reading in the three following strings:

- a) 000
- b) 0000
- c) 000000

2) What is the flaw in the following proof to show that if a language  $L$  is Turing recognizable, then we can create an enumerator to enumerate it? Remember that the sequence  $s_1, s_2, s_3, \dots$ , is an enumerated list of all strings in  $\Sigma^*$ , from shortest to longest, in lexicographical order for all strings of the same length.

Let  $M$  be a Turing machine that recognizes  $L$ .

We can create an enumerator  $E$  for  $L$  as follows:

- 1. Repeat the following for  $i = 1, 2, 3, \dots$
- 2. Run  $M$  on  $s_i$ .
- 3. If it accepts, print out  $s_i$ .

3) A Turing machines with a stay option is similar to an ordinary Turing machine except that the transition function has the form:

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$$

If  $\delta(q, a) = (r, b, S)$ , when the machine is in state  $q$  reading an  $a$ , the machine's head stays exactly where it is. Show that Turing machines with a stay option recognize the class of Turing-recognizable languages.

4) Show that Turing-decidable languages are closed under the following operations:

- a) union
- b) intersection
- c) complementation
- d) concatenation

5) Show that Turing-recognizable languages are closed under union and intersection. Why is it necessary to be more clever with these two proofs than those in question number 3?