

Weekly Proof Questions (Section 1.2)

Assigned: 1/15/2015

Due: 1/22/2015

1) Prove that every NFA can be converted to another equivalent NFA that has only one accept state.

2) Create NFAs to accept the following two languages:

$$L_1 = \{w \mid w \text{ begins with a 1 and ends with a 0.}\}$$

$$L_2 = \{w \mid w \text{ ends in 0101}\}$$

Justify why your NFAs are correct. (Namely, explain why it accepts all strings in the language and rejects all strings not in the language.) Furthermore, explain why it's easier to construct an NFA for these languages as opposed to a DFA.

3) For any string $w = w_1w_2w_3\dots w_n$, the reverse of w , written w^R , is the string w in reverse order, $w_nw_{n-1}\dots w_1$. For any language A , let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, so is A^R .

4) Draw a NFA that accepts the following language:

$$\{w \mid w \text{ contains exactly 3 0s after the last 1.}\}$$

5) Use the construction proof in the text that shows that the concatenation of two regular languages is regular to create an NFA that accepts the language L defined below.

$$L_1 = \{w \mid w \text{ ends in 01}\}$$

$$L_2 = \{w \mid w \text{ contains exactly 3 0s}\}$$

$$L = L_1L_2.$$

6) Your friend Tommy thinks that if he swaps the accept and reject states in an NFA that accepts a language L , that the resulting NFA must accept the language \bar{L} . Show, by way of counter-example, that Tommy is incorrect. Explain why your counter-example is one.

7) Let $\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}$$

Show that B is regular. (Note: Just prove that B^R is regular and it follows that B is as well, based on the proof shown in class.)