Weekly Proof Questions (Section 7.5)
Assigned: 4/16/2015
Due: 4/23/2015

1) The independent set problem is as follows: Given a graph G and an integer k, determine whether or not there are k vertices in G such that no two vertices out of the k share the same edge. Prove that the set INDEPENDET-SET is NP-Complete via a reduction from a known NP-Complete problem.

2) Let SET-SPLITTING = {<S,C> | S is a finite set and C = {C_1 , C_2 , C_3 , … , C_k}} is a collection of subsets of S, for some k > 0, such that the elements of S can be colored red or blue so that no C_i has all of its elements colored with the same color. Show that SET-SPLITTING is NP-Complete. (Hint: Reduce from NAE-SAT to SET-SPLITTING.)

3) n people live in a house and wish to share their expenses equally. Their respective expenses before settling are x_1, x_2, …, x_n. Assume that all of these are greater than 0. They agree to write each other checks so as to make each person's expenses equal the average cost. Naturally, they want to minimize the number of checks written. Formalize this as a decision problem and prove that it is NP-Complete.

4) Let HALF-CLIQUE = { <G> | G is an undirected graph having a complete subgraph with at least n/2 nodes, where n is the number of nodes in G }.

Show that HALF-CLIQUE is NP-complete.

5) Let SUBSET-SUM-k = { <S, t, k> | S is a set of positive integers, such that there exists a subset B of S of size k, such that the sum of the elements in B is equal to t, the target.}

Prove that SUBSET-SUM-k is NP-Complete by reducing SUBSET-SUM to it.