

Weekly Proof Questions (Section 7.4)

Assigned: 4/9/2015

Due: 4/16/2015

1) Let $\text{MAX-CLIQUE} = \{ \langle G, k \rangle \mid G \text{ is a graph and its largest clique is of size } k \}$. If CLIQUE is in P , prove that MAX-CLIQUE is ALSO in P . (Namely, given a black box that solves the CLIQUE decision problem in polynomial time, design a solution to MAX-CLIQUE in polynomial time.)

2) Why are 2 x 3 windows necessary in the proof of the Cook-Levin theorem?

3) Using the polynomial time reduction show in the text in 7.4, create the graph that the reduction produces for the following boolean formula:

$$(\bar{a} \vee b \vee \bar{c}) \wedge (a \vee \bar{b} \vee c) \wedge (\bar{a} \vee \bar{b} \vee c) \wedge (a \vee \bar{b} \vee \bar{c})$$

4) Show a polynomial time reduction from 4-SAT to 3-SAT, where 4-SAT represents satisfiability for boolean formulas with four literals in each clause instead of 3, and the formula is still in conjunctive normal form. (Namely, show how to transform a boolean formula in 4-SAT form into an equivalent formula in 3-SAT form such that the input formula is satisfiable if and only if the output formula is.)

5) Let $\text{DOUBLE-SAT} = \{ \langle \phi \rangle \mid \phi \text{ has at least two satisfying assignments} \}$. Show that DOUBLE-SAT is NP-Complete by giving a reduction from 3-SAT to DOUBLE-SAT .