COT 4210 Homework #2: Context Free Grammars, Turing Machines, Decidability
Due Date: Tuesday March 19, 2013 (in class)

1) Use the pumping lemma to show that the following language is not context free:

$\{0^n\#0^{2n}\#0^{3n} \mid n \geq 0\}$

2) Use the pumping lemma to show that the following language is not context free:

$\{0^n \mid n \in \text{Primes}\}$

3) Show that Turing-decidable languages are closed under the following operations:

a) union
b) intersection
c) complementation
d) concatenation

4) Show that Turing-recognizable languages are closed under union and intersection. Why is it necessary to be more clever with these two proofs than those in question number 3?

5) Find a match in the following instance of the PCP: $\{\left[\frac{ab}{abab}\right], \left[\frac{b}{a}\right], \left[\frac{aba}{b}\right], \left[\frac{aa}{a}\right]\}$.

6) Show that A_{TM} is not mapping reducible to E_{TM}.

7) Let S = { <M> | M is a TM that accepts w^R whenever it accepts w}. Show that S is undecidable.

8) Consider the problem of testing whether a Turning machine M on an input w ever attempts to move its head left at any point during its computation on w. Formulate this problem as a language and show that it's decidable.

9) Show that the PCP is decidable over a unary alphabet, that is, over the alphabet $\Sigma = \{1\}$.

10) Show that the PCP is undecidable over a binary alphabet, that is, over the alphabet $\Sigma = \{0, 1\}$. Note: In class we proved that the PCP was undecidable over a larger, fixed sized alphabet. You may use this result in your proof.