COT 4210 Homework #1: Regular Languages
Due Date: Thursday January 24, 2013 (in class)

Note: For all questions assume the alphabet is {0, 1}.

1) Draw the state diagram for the DFA formally described below:

$\{Q, \Sigma, \delta, q_0, F\}$ where

$Q = \{q_0, q_1, q_2, q_3\}$
$\Sigma = \{0, 1\}$
Start state = q_0
$F = \{q_1, q_3\}$
$\delta =$

	0	1
q_0	q_0	q_1
q_1	q_2	q_0
q_2	q_3	q_2
q_3	q_0	q_1

2) Draw a DFA that accepts the following language:

{ w | w's decimal equivalent is divisible by 5 }

3) Draw a DFA that accepts the following language:

{ w | w contains an odd number of 1s, or exactly 2 0s. }

4) Draw a NFA that accepts the following language:

{ w | w contains exactly 3 0s after the last 1. }

5) Use the construction proof in the text that shows that the concatenation of two regular languages is regular to create an NFA that accepts the language L defined below.

L_1 = { w | ends in 01 }
L_2 = { w | contains exactly 3 0s }
$L = L_1L_2$.

6) Your friend Tommy thinks that if he swaps the accept and reject states in an NFA that accepts a language L, that the resulting NFA must accept the language $\overline{L}$. Show, by way of counter-example, that Tommy is incorrect. Explain why your counter-example is one.

7) Let $\Sigma_3 = \{\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \ldots, \begin{bmatrix}1\\1\\1\end{bmatrix}\}$. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$B = \{ \in \Sigma_3^* |$ the bottom row of w is the sum of the top two rows $\}$

Show that B is regular. (Note: Just prove that B^R is regular and it follows that B is as well, based on the proof shown in class.)

8) Give a regular expression generating the following languages:

L_1 = {w | w begins with a 1 and ends with a 0.}
L_2 = {w | w contains the substring 0101}
L_3 = {w | the length of w does not exceed 5}
L_4 = {w | every odd position of w is a 1}
L_5 = { w | w contains an odd number of 1s, or exactly 2 0s. }

9) Use the algorithm described in class to convert a DFA to a regular expression on the two following DFAs described below:

DFA D_1: Q = {1, 2}, Σ = {a, b}, F = {2}, 1 is the start state, and δ is described as follows:

Q	Σ	Q
1	a	1
1	b	2
2	a	2
2	b	1

DFA D_2: Q = {1, 2, 3}, Σ = {a, b}, F = {1, 3}, 1 is the start state, and δ is described as follows:

Q	Σ	Q
1	a	2
1	b	2
2	a	2
2	b	3
3	a	1
3	b	2

10) Prove that the following languages are not regular via the Pumping Lemma:

L_1: $\{0^n1^n2^n \mid n \geq 0\}$
L_2: $\{ww \mid w \in \Sigma^*\}$
L_3: $\{a^{2^n} \mid n \geq 0\}$

11) Determine, with proof, whether or not the following languages are regular:

L_1: {w | contains the same number of occurrences of 01 as 10}
L_2: {w | contains the same number of occurrences of 00 as 11}

12) Determine a DFA with the minimum number of states that is equivalent to the DFA described below:

DFA D: Q = {0, 1, 2, 3, 4, 5, 6}, Σ = {a, b}, F = {1,3,5,6}, 0 is the start state, and δ is described as follows:

Q	Σ	Q
0	a	1
0	b	3
1	a	2
1	b	4
2	a	5
2	b	5
3	a	4
3	b	2
4	a	5
4	b	5
5	a	6
5	b	5
6	a	6
6	b	6

Please use the algorithm shown in class.