

COT 4210 Proof Homework #2: Classes P and NP
Due Date: April 10, 2012 in class

- 1) A 5-pointed-star in an undirected graph is a 5-clique. Show that 5-POINTED-STAR \in P, where 5-POINTED-STAR = { $\langle G \rangle$ | G contains a 5-pointed-star as a subgraph }.
- 2) Let CONNECTED = { $\langle G \rangle$ | G is a connected unweighted graph }. Show that CONNECTED \in P.
- 3) Let 2-SAT = { $\langle \phi \rangle$ | ϕ is a satisfiable 2-CNF formula }. (2-CNF is the same as 3-CNF, but with only 2 variables allowed per clause.) Show that 2-SAT \in P.
- 4) Let DOUBLE-SAT = { $\langle \phi \rangle$ | ϕ has at least two satisfying assignments }. Show that DOUBLE-SAT is NP-Complete by giving a reduction from 3-SAT to DOUBLE-SAT.
- 5) Let HALF-CLIQUE = { $\langle G \rangle$ | G is an undirected graph having a complete subgraph with at least $n/2$ nodes, where n is the number of nodes in G }.

Show that HALF-CLIQUE is NP-complete.

- 6) Let SUBSET-SUM- k = { $\langle S, t, k \rangle$ | S is a set of positive integers, such that there exists a subset B of S of size k , such that the sum of the elements in B is equal to t , the target. }

Prove that SUBSET-SUM- k is NP-Complete by reducing SUBSET-SUM to it.

- 7) Let SET-SPLITTING = { $\langle S, C \rangle$ | S is a finite set and $C = \{C_1, C_2, \dots, C_k\}$ is a collection of subsets of S , for some $k > 0$, such that the elements of S can be colored red or blue so that no C_i has all of its elements colored with the same color. } Show that SET-SPLITTING is NP-complete.