COT 4210 Homework #1: Regular Languages
Due Date: Tuesday January 24, 2012

Note: For all questions assume the alphabet is {0, 1}.

1) Draw the state diagram for the DFA formally described below:

$\{Q, \Sigma, \delta, q_0, F\}$ where

$Q = \{q_0, q_1, q_2, q_3\}$
$\Sigma = \{0, 1\}$
Start state = q_0
$F = \{q_1, q_3\}$
$\delta =$

	0	1
q_0	q_0	q_1
q_1	q_2	q_0
q_2	q_3	q_2
q_3	q_0	q_1

2) Draw a DFA that accepts the following language:

{ w | w's decimal equivalent is divisible by 5 }

3) Draw a DFA that accepts the following language:

{ w | w contains an odd number of 1s, or exactly 2 0s. }

4) Draw a NFA that accepts the following language:

{ w | w contains exactly 3 0s after the last 1. }

5) Use the construction proof in the text that shows that the concatenation of two regular languages is regular to create an NFA that accepts the language L defined below.

L_1 = { w | ends in 01 }
L_2 = { w | contains exactly 3 0s }
$L = L_1L_2$.

6) Prove that every NFA can be converted to another equivalent NFA that has only one accept state.

7) Your friend Tommy thinks that if he swaps the accept and reject states in an NFA that accepts a language L, that the resulting NFA must accept the language $\overline{L}$. Show, by way of counter-example, that Tommy is incorrect. Explain why your counter-example is one.

8) For any string $w = w_1w_2w_3\dots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_nw_{n-1}\dots w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, so is A^R.

9) Let $\Sigma_3 = \left\{\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \dots, \begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$B = \{ \in \Sigma_3^* \mid \text{the bottom row of w is the sum of the top two rows} \}$$

Show that B is regular. (Note: Just prove that B^R is regular and it follows that B is as well, based on the proof shown in class.)

10) Give a regular expression generating the following languages:

$L_1 = \{w \mid w \text{ begins with a 1 and ends with a 0.}\}$
$L_2 = \{w \mid w \text{ contains the substring 0101}\}$
$L_3 = \{w \mid \text{the length of w does not exceed 5}\}$
$L_4 = \{w \mid \text{every odd position of w is a 1}\}$
$L_5 = \{ w \mid w \text{ contains an odd number of 1s, or exactly 2 0s.} \}$

11) Use the procedure shown in class and shown in the textbook to convert the following regular expressions to NFAs:

a. $(0 \cup 1)^*000(0 \cup 1)^*$
b. $(((00)^*(11)) \cup 01)^*$

12) Use the algorithm described in class to convert a DFA to a regular expression on the two following DFAs described below:

DFA D_1: Q = {1, 2}, Σ = {a, b}, F = {2}, 1 is the start state, and δ is described as follows:

Q	Σ	Q
1	a	1
1	b	2
2	a	2
2	b	1

DFA D_2: Q = {1, 2, 3}, Σ = {a, b}, F = {1, 3}, 1 is the start state, and δ is described as follows:

Q	Σ	Q
1	a	2
1	b	2
2	a	2
2	b	3
3	a	1
3	b	2

13) Prove that the following languages are nor regular via the Pumping Lemma:

L_1: $\{0^n 1^n 2^n \mid n \geq 0\}$
L_2: $\{ww \mid w \in \Sigma^*\}$
L_3: $\{a^{2^n} \mid n \geq 0\}$

14) Determine, with proof, whether or not the following languages are regular:

L_1: {w | contains the same number of occurrences of 01 as 10}
L_2: {w | contains the same number of occurrences of 00 as 11}

15) Determine a DFA with the minimum number of states that is equivalent to the DFA described below:

DFA D: Q = {0, 1, 2, 3, 4, 5, 6}, Σ = {a, b}, F = {1,3,5,6}, 0 is the start state, and δ is described as follows:

Q	Σ	Q
0	a	1
0	b	3
1	a	2
1	b	4
2	a	5
2	b	5
3	a	4
3	b	2
4	a	5
4	b	5
5	a	6
5	b	5
6	a	6
6	b	6

Please use the algorithm shown in class.