Fall 2024 COT 4210 Homework #6: Class P, Class NP

1) A 5-pointed-star in an undirected graph is a 5-clique. Show that 5-POINTED-STAR ∈ P, where 5-POINTED-STAR = { <G> | G contains a 5-pointed-star as a subgraph }.

2) Let 2-SAT = { <φ> | φ is a satisfiable 2-CNF formula }. (2-CNF is the same as 3-CNF, but with only 2 variables allowed per clause.) Show that 2-SAT ∈ P.

3) Let MODEXP = { <a,b,c,p> | a, b, c, and p are binary integers such that $a^b \equiv c \pmod{p}$ }. Prove that MODEXP ∈ P.

4) Let LCS = { <a, b, c> | a, b are strings and c (represented in binary) is a non-negative integer such that the longest common subsequence between a and b is of length c. } Prove that LCS ∈ P.

5) Why are 2 x 3 windows necessary in the proof of the Cook-Levin theorem?

6) Using the polynomial time reduction show in the text in 7.4 from 3-SAT to CLIQUE, create the graph that the reduction produces for the following Boolean formula:

$$(\bar{a} \vee b \vee \bar{c}) \wedge \left(a \vee \bar{b} \vee c\right) \wedge \left(\bar{a} \vee \bar{b} \vee c\right) \wedge (a \vee \bar{b} \vee \bar{c})$$

7) Let HALF-CLIQUE = { <G> | G is an undirected graph having a complete subgraph with at least n/2 nodes, where n is the number of nodes in G }.

Show that HALF-CLIQUE is NP-complete.

8) Let MAX-CLIQUE = { <G, k> | G is a graph and its largest clique is of size k}. If CLIQUE is in P, prove that MAX-CLIQUE is ALSO in P. (Namely, given a black box that solves the CLIQUE decision problem in polynomial time, design a solution to MAX-CLIQUE in polynomial time.

9) Show a polynomial time reduction from 4-SAT to 3-SAT, where 4-SAT represents satisfiability for boolean formulas with four literals in each clause instead of 3, and the formula is still in conjunctive normal form. (Namely, show how to transform a boolean formula in 4-SAT form into an equivalent formula in 3-SAT form such that the input formula is satisfiable if and only if the output formula is.)

10) The independent set problem is as follows: Given a graph G and an integer k, determine whether or not there are k vertices in G such that no two vertices out of the k share the same edge. Prove that the set INDEPENDET-SET is NP-Complete via a reduction from a known NP-Complete problem.

11) n people live in a house and wish to share their expenses equally. Their respective expenses before settling are x_1, x_2, …, x_n. Assume that all of these are greater than 0. They agree to write each other checks so as to make each person's expenses equal the average cost. Naturally, they want to minimize the number of checks written. Formalize this as a decision problem and prove that it is NP-Complete.