

COT 4210 Homework #4: Sections 2.1, 2.2
Assigned: 9/19/2024
Due Date/Time: On Webcourses

1) Give parse trees and derivations for the following two strings

- a) $a \times (a + a)$
- b) $a \times a + a$

in the CFG G_4 defined below:

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T \times F \mid F \\ F &\rightarrow (E) \mid a \end{aligned}$$

2) Give context free grammars that generates the following languages with the alphabet $\{0,1\}$:

- a) $\{0^n 1^m \mid n > 2m \geq 0\}$
- b) $\{w \mid w \text{ contains the same number of 0s and 1s in any order}\}$

For each grammar, briefly justify why it generates the desired language.

3) Give an informal description and state diagram of a PDA that accepts the languages in question 2. (So draw the full diagram and explain the gist of how it works in English.)

4) Convert the following CFG (with start variable S and alphabet $\{0,1\}$) into an equivalent CFG in Chomsky normal form, using the algorithm shown in class.

$$\begin{aligned} S &\rightarrow 1S \mid BA \\ A &\rightarrow BAB \mid B \mid 111 \\ B &\rightarrow 00 \mid \varepsilon \end{aligned}$$

5) Show that context-free languages are NOT closed under intersection by proving that both languages below are Context Free, but that their intersection is not:

$$\begin{aligned} A &= \{a^m b^n c^n \mid m, n \geq 0\} \\ B &= \{a^n b^n c^m \mid m, n \geq 0\} \end{aligned}$$

Question: Is the union of these languages Context-Free?

6) Show that if G is a CFG in Chomsky Normal Form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n-1$ steps are required for any derivation of w .

7) Use the pumping lemma for context free grammars to prove that the language of all palindromes over the alphabet $\{0,1\}$ with an equal number of 0s and 1s is not context free.