

COT 4210 Homework #5: Classes P and NP
Due Date: Wednesday, July 27nd, 2011 in class

1) A triangle in an undirected graph is a 3-clique. Show that $\text{TRIANGLE} \in P$, where $\text{TRIANGLE} = \{ \langle G \rangle \mid G \text{ contains a triangle} \}$.

2) Let $\text{DOUBLE-SAT} = \{ \langle \phi \rangle \mid \phi \text{ has at least two satisfying assignments} \}$. Show that DOUBLE-SAT is NP-Complete by giving a reduction from 3-SAT to DOUBLE-SAT .

3) Let $\text{HALF-CLIQUE} = \{ \langle G \rangle \mid G \text{ is an undirected graph having a complete subgraph with at least } n/2 \text{ nodes, where } n \text{ is the number of nodes in } G \}$.

Show that HALF-CLIQUE is NP-complete.

4) Let $\text{SUBSET-SUM-}k = \{ \langle S, t, k \rangle \mid S \text{ is a set of positive integers, such that there exists a subset } B \text{ of } S \text{ of size } k, \text{ such that the sum of the elements in } B \text{ is equal to } t, \text{ the target.} \}$

Prove that $\text{SUBSET-SUM-}k$ is NP-Complete by reducing SUBSET-SUM to it.

5) Optional Program (This won't be graded, but the input data and correct output will be posted so that you can check to see if your program works.) Write a program that reads its input from the file "3sat.txt" and outputs to a file called "subsetsum.txt". The job of your program will be to convert a 3-SAT instance into a SUBSET-SUM instance such that the former is in 3-SAT iff the latter is in SUBSET-SUM. Make sure to hard-code the file names so your program can be immediately run without any user input. Make sure you match the sample shown below. Here are the file formats for both problems:

3-SAT file format

The first line contains a single positive integer, X , representing the number of problems to solve. The problems follow. The first line of each problem to solve contains two positive integers, n and v , separated by a space. n is the number of clauses in the 3-SAT expression and v is the number of variables in the 3-SAT expression. The next n lines of the file will contain one clause of the 3-SAT expression each. Each line of these subsequent lines contains three integers separated by spaces. All integers will be either in between 1 and v , inclusive or $-v$ and -1 , inclusive. In particular, a positive integer k stands for the variable x_k , while the negative integer $-k$ stands for the negation of the variable x_k . As an example, the clauses:

$$(x_1 \vee \sim x_2 \vee x_3) \wedge (\sim x_1 \vee x_2 \vee x_3) \wedge (\sim x_1 \vee \sim x_2 \vee \sim x_3) \wedge (\sim x_1 \vee \sim x_2 \vee x_3) \text{ and } (x_1 \vee \sim x_1 \vee x_2)$$

would be stored in a file as follows:

```
2
4 3
1 -2 3
-1 2 3
-1 -2 -3
-1 -2 3
1 2
1 -1 2
```

SUBSET-SUM file format

The first line of output for each case contains two integers, n and t , where n is the number of integers in the subset and t is the target. The second line of output for each case contains n integers that comprise the set, separated by spaces. Just follow the output for each case, line by line. If there are X cases in the input file, there should be $2X$ lines of output in the output file.

Here is a sample file, that corresponds to the 3-SAT file example above:

```
14 5631
4160 4117 1040 1093 337 260 64 64 16 16 4 4 1 1
6 23
17 17 5 4 1 1
```

Note: Use the construction at the end of section 7.5, except determine the values of the numbers in base 4 instead of base 10.