COT 4210 Homework #4: Decidability
Due Date: Wednesday July 13, 2011 (in class)

1) Let ALL_{DFA} = { <A> | A is a DFA that recognizes Σ^*}. Show that ALL_{DFA} is decidable.

2) Let $INFINITE_{DFA}$ = { <A> | A is a DFA and L(A) contains an infinite number of strings}. Show that $INFINITE_{DFA}$ is decidable.

Note: The next two questions are programming questions. Please just attach your code to your written homework. We won't actually run your code, but are simply looking for the thinking behind it. But, make sure you run it so (a) you know it works, (b) you can see the tangible result based on this theoretical material.

3) Write an enumerator that prints out all the fractions (in lowest terms) with denominators less than or equal to 1000, in between 0 and 1, not including either value. A gcd function will be useful. In addition to making your program a valid enumerator, make sure it does not output any two fractions that are equivalent in value. (Note: Points will be taken off if fractions are repeated or if two different fractions with the same value are outputted.)

4) Define the halting problem as follows:

$HALT_{TM}$ = { <M, w> | M runs and halts (either accepts or rejects) when run on w. }

This language is NOT Turing decidable, but it is Turing recognizable. A very similar proof to the one shown in class to prove that A_{TM} is not decidable can be used to prove the same for $HALT_{TM}$. Give a programming illustration of this proof similar to the code handout given in class on Tuesday, 10/12. (Note: Your program might just run forever!)

5) Consider the problem of testing whether a Turning machine M on an input w ever attempts to move its head left at any point during its computation on w. Formulate this problem as a language and show that it's decidable.

6) Show that the PCP is decidable over a unary alphabet, that is, over the alphabet $\Sigma = \{1\}$.

7) Find a match in the following instance of the PCP: $\{\left[\frac{ab}{abab}\right], \left[\frac{b}{a}\right], \left[\frac{aba}{b}\right], \left[\frac{aa}{a}\right]\}$.