

**COT 4210 Homework #3: Turing Machines**  
**Due Date: Friday June 24, 2011 (to HEC-240 or Arup's mailbox in HEC-346)**

**1)** Give the sequence of configurations that the Turing machine  $M_2$  in chapter 3 of the text (and shown in class) goes through while reading in the three following strings:

- a) 000
- b) 0000
- c) 000000

**2)** What is the flaw in the following proof to show that if a language  $L$  is Turing recognizable, then we can create an enumerator to enumerate it? Remember that the sequence  $s_1, s_2, s_3, \dots$ , is an enumerated list of all strings in  $\Sigma^*$ , from shortest to longest, in lexicographical order for all strings of the same length.

Let  $M$  be a Turing machine that recognizes  $L$ .

We can create an enumerator  $E$  for  $L$  as follows:

1. Repeat the following for  $i = 1, 2, 3, \dots$
2. Run  $M$  on  $s_i$ .
3. If it accepts, print out  $s_i$ .

**3)** Let a 2-PDA be a pushdown automata with access to 2 stacks. (In each transition, we can read the top of both stacks and push something on top of both stacks, if we choose.)

- a) Give an example of a language that is NOT context free that can be accepted by a 2-PDA. Briefly describe in words how this 2-PDA would operate to accept that language.
- b) Show that a standard Turing Machine can be implemented using a 2-PDA.

**4)** A Turing machines with a stay option is similar to an ordinary Turing machine except that the transition function has the form:

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$$

If  $\delta(q, a) = (r, b, S)$ , when the machine is in state  $q$  reading an  $a$ , the machine's head stays exactly where it is. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

**5)** Show that Turing-decidable languages are closed under the following operations:

- a) union
- b) intersection
- c) complementation
- d) concatenation

**6)** Show that Turing-recognizable languages are closed under union and intersection. Why is it necessary to be more clever with these two proofs than those in question number 5?

**7)** Explain the trouble in showing that Turing-recognizable languages are closed under complementation.