COT 4210 (Proof) Homework #2: Turing Machines
Due Date: Tuesday October 11, 2011 (in class)

1) Give the sequence of configurations that the Turing machine M_2 in chapter 3 of the text (and shown in class) goes through while reading in the three following strings:

a) 000
b) 0000
c) 000000

2) Write a formal Turing Machine program on the applet linked here:

http://math.hws.edu/TMCM/java/labs/xTuringMachineLab.html#applet

which halts only if the input string takes the following form

AzB, where $A \in (x \cup y)^*$, $B \in (x \cup y)^*$, and A and B contain the same number of x's.

Examples of strings on which the program should halt are: xxyzzz, z, yyyyyyyyz, and xzx.
Examples of strings on which the program should loop are: xxyxx, xzxxzx. Yyyxzyxx.

3) What is the flaw in the following proof to show that if a language L is Turing recognizable, then we can create an enumerator to enumerate it? Remember that the sequence s_1, s_2, s_3, …, is an enumerated list of all strings in Σ^*, from shortest to longest, in lexicographical order for all strings of the same length.

Let M be a Turing machine that recognizes L.

We can create an enumerator E for L as follows:

1. Repeat the following for i = 1, 2, 3, …
2. Run M on s.
3. If it accepts, print out s_i.

4) Let a 2-PDA be a pushdown automata with access to 2 stacks. (In each transition, we can read the top of both stacks and push something on top of both stacks, if we choose.)

a) Give an example of a language that is NOT context free that can be accepted by a 2-PDA. Briefly describe in words how this 2-PDA would operate to accept that language.

b) Show that a standard Turing Machine can be implemented using a 2-PDA.

5) A Turing machines with a stay option is similar to an ordinary Turning machine except that the transition function has the form:

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$$

If δ(q, a) = (r, b, S), when the machine is in state q reading an a, the machine's head stays exactly where it is. Show that Turing machines with a stay option recognize the class of Turing-recognizable languages.

6) Show that Turing-decidable languages are closed under the following operations:

a) union
b) intersection
c) complementation
d) concatenation

7) Show that Turing-recognizable languages are closed under union and intersection. Why is it necessary to be more clever with these two proofs than those in question number 5?

8) Explain the trouble in showing that Turing-recognizable languages are closed under complementation.