

**COT 4210 Homework #8: Classes P and NP**  
**Due Date: December 2<sup>nd</sup>, 2010 in class**

- 1) A triangle in an undirected graph is a 3-clique. Show that  $\text{TRIANGLE} \in \text{P}$ , where  $\text{TRIANGLE} = \{ \langle G \rangle \mid G \text{ contains a triangle} \}$ .
- 2) Let  $\text{HALF-CLIQUE} = \{ \langle G \rangle \mid G \text{ is an undirected graph having a complete subgraph with at least } n/2 \text{ nodes, where } n \text{ is the number of nodes in } G \}$ .

Show that  $\text{HALF-CLIQUE}$  is NP-complete.

- 3) A coloring of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Let

$\text{3-COLOR} = \{ \langle G \rangle \mid \text{the nodes of } G \text{ can be colored with three colors such that no two nodes joined by an edge have the same color.} \}$

Show that  $\text{3COLOR}$  is NP-Complete. (Note: There is an extra hint in the textbook for this problem. Please try to use that hint.)

- 4) Let  $\text{SUBSET-SUM-}k = \{ \langle S, t, k \rangle \mid S \text{ is a set of positive integers, such that there exists a subset } B \text{ of } S \text{ of size } k, \text{ such that the sum of the elements in } B \text{ is equal to } t, \text{ the target.} \}$

Prove that  $\text{SUBSET-SUM-}k$  is NP-Complete by reducing  $\text{SUBSET-SUM}$  to it.