

**COT 4210 Homework #4: Turing Machines**  
**Due Date: Tuesday October 12, 2010 (in class)**

**1)** Give the sequence of configurations that the Turing machine  $M_2$  in chapter 3 of the text (and shown in class) goes through while reading in the three following strings:

- a) 000
- b) 0000
- c) 000000

**2)** What is the flaw in the following proof to show that if a language  $L$  is Turing recognizable, then we can create an enumerator to enumerate it? Remember that the sequence  $s_1, s_2, s_3, \dots$ , is an enumerated list of all strings in  $\Sigma^*$ , from shortest to longest, in lexicographical order for all strings of the same length.

Let  $M$  be a Turing machine that recognizes  $L$ .

We can create an enumerator  $E$  for  $L$  as follows:

1. Repeat the following for  $i = 1, 2, 3, \dots$
2. Run  $M$  on  $s_i$ .
3. If it accepts, print out  $s_i$ .

**3)** Let a 2-PDA be a pushdown automata with access to 2 stacks. (In each transition, we can read the top of both stacks and push something on top of both stacks, if we choose.)

- a) Give an example of a language that is NOT context free that can be accepted by a 2-PDA. Briefly describe in words how this 2-PDA would operate to accept that language.
- b) Show that a standard Turing Machine can be implemented using a 2-PDA.

**4)** A Turing machines with left reset is similar to an ordinary Turning machine except that the transition function has the form:

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, \text{RESET}\}$$

If  $\delta(q, a) = (r, b, \text{RESET})$ , when the machine is in state  $q$  reading an  $a$ , the machine's head jumps to the left-hand end of the tape after it writes  $b$  in the tape and enters state  $r$ . Note that these machines don't have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

**5)** Show that Turing-decidable languages are closed under the following operations:

- a) union
- b) intersection
- c) complementation
- d) concatenation

**6)** Show that Turing-recognizable languages are closed under union and intersection. Why is it necessary to be more clever with these two proofs than those in question number 5?

**7)** Explain the trouble in showing that Turing-recognizable languages are closed under complementation.

**Extra Credit:** Write your own TM simulator. Your simulator should read in the description of a TM from a file (prompt the user for the input file name), and then as the user for an input string. Then your program should output whether that string gets accepted or rejected by the machine. (If neither occurs, your program does not need to detect the loop!!!) With your submission, include details of how to create an input file. Make sure you accommodate for both an accept and reject state, as well as how the book describes moving off the leftmost tape square. I will accept this extra credit any time this semester before the last homework assignment is due. If you wish, you can write the simulator to be a GUI or an applet that visually shows each transition, but this is NOT necessary to get the extra credit.