COT 4210 Homework #1: DFAs and NFAs
Due Date: Thursday September 2, 2010

Note: For all questions assume the alphabet is {0, 1}.

1) Draw the state diagram for the DFA formally described below:

$\{Q, \Sigma, \delta, q_0, F\}$ where

$Q = \{q_0, q_1, q_2, q_3\}$
$\Sigma = \{0, 1\}$
Start state $= q_0$
$F = \{q_1, q_3\}$
$\delta =$

	0	1
q_0	q_0	q_1
q_1	q_2	q_0
q_2	q_3	q_2
q_3	q_0	q_1

2) Draw a DFA that accepts the following language:

{ w | w's decimal equivalent is divisible by 5 }

3) Draw a DFA that accepts the following language:

{ w | w contains an odd number of 1s, or exactly 2 0s. }

4) Draw a NFA that accepts the following language:

{ w | w contains exactly 3 0s after the last 1. }

5) Use the construction proof in the text that shows that the concatenation of two regular languages is regular to create an NFA that accepts the language L defined below.

$L_1 = \{$ w | ends in 01 $\}$
$L_2 = \{$ w | contains exactly 3 0s $\}$
$L = L_1 L_2$.

6) Prove that every NFA can be converted to another equivalent NFA that has only one accept state.

7) Your friend Tommy thinks that if he swaps the accept and reject states in an NFA that accepts a language L, that the resulting NFA must accept the language $\bar{L}$. Show, by way of counter-example, that Tommy is incorrect. Explain why your counter-example is one.

8) For any string $w = w_1w_2w_3\ldots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_nw_{n-1}\ldots w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, so is A^R.

9) Let $\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$. A string of symbols in Σ_3 gives three rows of 0s and 1s.
Consider each row to be a binary number and let

$$B = \{ w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows} \}$$

Show that B is regular. (Note: Just prove that B^R is regular and it follows that B is as well, based on the proof shown in class.)