Recitation #9 Warm-Up Problems
3/14/2014

1) In the sequence 2001, 2002, 2003, ..., each term after the third is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is 2001 + 2002 - 2003 = 2000. What is the 2004^{th} term of the sequence?

2) Each face of a cube is painted either red or blue, each with probability 1/2. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

3) All the students in an algebra class took a 100-point test. Five students scored 100, each student scored at least 60, and the mean score was 76. What is the smallest possible number of students in the class?

4) If $f(x) = ax + b$ and $f^{-1}(x) = bx + a$, what are a and b?

5) The two digits in Jack's age are the same as the digits in Bill's age, but in reverse order. In five years Jack will be twice as old as Bill will be then. How old are Jack and Bill now?

Recitation #9 Induction Problems

1) Prove using induction on n for all non-negative integers n that

$$\sum_{i=0}^{n}\left(-\frac{1}{2}\right)^i = \frac{2^{n+1}+(-1)^n}{3(2^n)}$$

2) Packets of ramen at the Sam's Club are sold in sets of three for "Top Ramen" brand, and sets of four for "Myojo" brand. Use strong induction to prove that it is possible to buy any number of ramen packets greater than 11.

3) Let T be a sequence defined as follows: $T_0 = 1$, $T_1 = 1$, and $T_n = T_{n-1} + 2T_{n-2}$, for all integers n > 1. Use strong induction with two base cases to prove that $T_n = \frac{2^{n+1}}{3} + \frac{(-1)^n}{3}$, for all positive integers n.

4) Consider the following four equations:

a) 1 = 1
b) 2+3+4 = 1 + 8
c) 5+6+7+8+9 = 8 + 27
d) 10+11+12+13+14+15+16 = 27 + 64

Conjecture the general formula suggested by these four equations and use induction to prove your conjecture.

5) Suppose you begin with a pile of *n* stones and split this pile into *n* piles of one stone each by successively splitting a pile of stones into two smaller piles. Each time you split a pile you multiply the number of stones in each of the two smaller piles you form, so that if these piles have *r* and *s* stones respectively, you compute *rs*. Using strong induction on n, show that no matter how you split the piles, the sum of the products of all the steps equals n(n - 1)/2.