Recitation #8 Warm-Up Problems
2/28/2014

1) A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

2) Let f be a function with the following properties:

(i) f(1) = 1, and
(ii) f(2n) = nf(n), for any positive integer n

What is the value of $f(2^{500})$?

3) Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

4) What is the smallest integer x for which the expression $log_{1000}(log_{2000}(log_{3000}(log_{4000}x)))$ is defined?

5) If $\sum_{n=0}^{\infty} cos^{2n}\theta = 4$ and $0 < \theta < \frac{\pi}{2}$, what is θ?

Recitation 8: Induction Problems

1) Use mathematical induction on n to prove that $\gcd(F_{n+1}, F_n) = 1$, for all positive integers n, where F_n denotes the n^{th} Fibonacci number. (Remember $F_1 = F_2 = 1$, $F_n = F_{n-1}+F_{n-2}$, for n > 2.)

2) Use mathematical induction on n to prove that $5 \mid (n^5 - n)$, for all positive integers n. Note: Remember that $(n+1)^5 = n^5 + 5n^4 + 10n^3 + 10n^2 + 5n + 1$.

3) Let $H_n = \sum_{i=1}^{n} \frac{1}{i}$. Use mathematical induction on n to prove $\sum_{i=1}^{n} i = (n+1)H_n - n$, for all positive integers n.

4) Work out a few examples and attempt to conjecture the result of calculating $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}^n$, for all real numbers a and positive integers n, in terms of a and n. Prove your result using induction on n.

5) Let T(n) be a recurrence relation defined by T(1) = 2, T(n) = 2nT(n - 1), for all integer n > 1. Prove, using induction on n, that for all positive integers n, $T(n) = 2^n n!$.