Recitation #7 Warm-Up Problems
2/21/2014

1) The second and fourth terms of a geometric progression are 2 and 6, respectively? What are all of the possible values of the first term?

2) Find the value of x that satisfies the equation $25^{-2} = \frac{5^{\frac{48}{x}}}{(5^{\frac{26}{x}})(25^{\frac{17}{x}})}$.

3) There are 200 players in a single elimination tennis tournament. In a single elimination tournament, you are out of the tournament after your first loss. In the first round 56 players are given byes and the remaining 144 players are paired up to play against each other. Thus, after this initial round 56 + 72 = 128 players remain. From this point on, for all subsequent rounds, all the players are paired up to play, with half of them surviving into the next round. The rounds continue until there is a single winner. How many games were played in the tournament total? (We assume that each game ends in a win for one player and a loss for the other, ie. no ties.)

4) What is the largest integer that is a divisor of (n + 1)(n + 3)(n + 5)(n + 7)(n + 9), for all positive even integers n?

5) A bag contains two red beads and two green beads. You reach into the bag and pull out a bead, replacing it with a red bead regardless of the color you pulled out. What is the probability that all beads in the bad are red after three such replacements? (Assume that each bead is equally likely to be pulled whenever you reach into the bag.)

Recitation #7: Number Theory Problems

1) Find the greatest common divisor of 975 and 414 using Euclid's Algorithm.

2) Using the Extended Euclidean Algorithm, determine all sets of integers x and y, such that 325x + 138y = 1.

3) Using the Extended Euclidean Algorithm, determine all sets of integers x and y, such that 171x + 140y = 1.

4) Using Fermat's Little Theorem ($a^{p-1} \equiv 1 \pmod p$), for all integers a and primes p such that gcd(a, p) = 1), determine, without the use of any electronic device, the remainder when 324^{3601} is divided by 1801. Note: 1801 is a prime number.

5) Given a sequence S = $a_1, a_2, \ldots, a_n$, define f(S) = $\sum_{i=2}^{n} \gcd(\bigcup_{j=1}^{i} a_j)$. For example, if our sequence was 6, 18, and 4, then f(S) = gcd(6, 18) + gcd(6, 18, 4) = 6 + 2 = 8. Consider the following set T = {128, 96, 14, 105, 32, 17, 98, 72, 36, 24}. We can use this set to define a sequence by putting the values in T in any order. Define g(T) = max(f(perm(T)), where perm(T) is any permutation of T. Determine g(T) for this set and describe your intuitive strategy in arriving at this value. Note: The intuitive strategy that works for this case won't necessarily work in all cases. Also, in many cases, there are several possible permutations that all produce the maximum value, g(T).