Recitation #6 Warm-Up Problems
2/14/2014

1) Asha randomly selects two distinct numbers from the set {1, 2, 3, 4, 5}, and Vikram randomly selects a number from the set {1, 2, …, 10}. What is the probability that Vikram's number is greater than the sum of Asha's numbers?

2) Several sets of prime numbers, such as {7, 83, 421, 659}, use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?

3) Sarah pours four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then transfers half the coffee from the first cup to the second and, after stirring thoroughly, transfers half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

4) Chan leaves to go to work at 8 AM every morning. When he averages 40 miles per hour, he arrives 3 minutes late. When he averages 60 miles per hour, he arrives 3 minutes early. At what speed should he drive to arrive at his workplace exactly on time?

5) Suppose that a and b are digits, not both nine and not both zero, and the repeating decimal $0.\overline{ab}$ is expressed as a fraction in lowest terms. How many different denominators are possible?

Recitation #6: Number Theory Problems

2/14/2014

1) Prove that if $a \equiv b \pmod{n}$, then $a^k \equiv b^k \pmod{n}$. You may use the fact that if x and y are integers and k is a positive integer, then $(x + y)^k$ can be written as $Cx + y^k$, for some integer C.

2) Calculate $6^i \bmod 13$ for $0 \le i < 13$, by hand.

3) Using trial and error, find the smallest positive integer x such that $18x \equiv 1 \pmod{23}$.

4) Prime factorize 41173 by hand.

5) To prove there were an infinite number of primes, we utilized an expression of the form $p_1p_2\ldots p_n + 1$, where p_i denotes the i^{th} prime number. Determine the smallest value of n for which this expression isn't prime. Explain why this example DOESN'T negate the proof shown in class.