

Recitation #4 Warm-Up Problems
1/31/2014

- 1) How many positive integers b have the property that $\log_b 729$ is a positive integer?

- 2) Two non-zero real numbers, a and b , satisfy $ab = a - b$. What are all the possible values of $\frac{a}{b} + \frac{b}{a} - ab$?

- 3) Let A , M and C be non-negative integers such that $A + M + C = 12$. What is the maximum value of $AMC + AM + MC + AC$?

- 4) Let f be a function for which $f\left(\frac{x}{3}\right) = x^2 + x + 1$. $f(3z) = ax^2 + bz + c$. Determine the values of a , b and c .

- 5) A checkerboard of 13 rows and 17 columns has a number written in each square, beginning in the upper left corner, so that the first row is numbered 1, 2, 3, ..., 17, the second row 18, 19, 20, ..., 24, and so on down the board. If the board is renumbered so that the left column, top to bottom, is 1, 2, 3, ..., 13, the second column is 14, 15, 16, ..., 26 and so on across the board, some squares have the same number in both numbering systems. What is the sum of these squares?

Recitation #4: Set Problems

1/31/2014

- 1) Using set laws, prove the following: $(C - (A \cup B)) \cup (B \cap C) \cup (A \cap C) = C$.

- 2) Prove or disprove for arbitrary sets A , B and C :

If $C \subseteq B$, then, $(A - B) \cup (B - C) = \neg C \cap (A \cup B)$.

- 3) Prove the following for arbitrary sets A , B and C , using proof by contradiction.

If $(A - B) - C = A - (B - C)$ then $A \cap C = \emptyset$.