Recitation #2 Warm-Up Problems
1/17/2014

1) A speaker talked for 60 minutes to a full auditorium. Twenty percent of the audience heard the entire talk and ten percent slept through the entire talk. Half of the remainder of the audience heard one third of the talk and the other heard two thirds of the talk. What was the average number of minutes of the talk heard by the members of the audience?

2) Walter rolls 4 standard six-sided dice and finds that the product of the numbers on the upper faces is 144. What are all the possible sums of the upper four faces?

3) Let R be a rectangle. How many circles in the plane of R have a diameter both of whose endpoints are vertices of R?

4) How many different prime numbers are factors of N if

$$log_2\left(log_3\left(log_5(log_7 N)\right)\right) = 11?$$

What are those prime numbers?

5) If $2^{1998} - 2^{1997} - 2^{1996} + 2^{1995} = k(2^{1995})$, what is the value of k?

Recitation #2: Proof Problems
1/17/2014

For questions 1 and 2, use the Rules of Inference and the Law of Contraposition to validate the conclusion drawn below. (Each of the items above the dotted line is a premise, while the conclusion to draw is below the dotted line.) Show each step and state which rule is being used.

1) Prove the desired conclusion using the premises shown below.

$p \vee q$
$\neg r \rightarrow \neg p$
$r \rightarrow s$
$\neg q$
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s

2) Prove the desired conclusion using the premises shown below.

$q \rightarrow \neg t$
$p \rightarrow q$
$\neg r \rightarrow (p \wedge q)$
$(p \wedge q) \rightarrow \neg p$

$\neg p \vee (\neg t \wedge r)$

3) Prove that all perfect squares (numbers that can be written as k^2 for some integer k) have an odd number of divisors. Extend the argument to show that all non-perfect squares (all other positive integers) have an even number of divisors. (Hint: Note that most divisors come in pairs. As an example, twelve's divisors are the following pair – (1, 12), (2, 6), (3, 4).)

4) At a party, each person counts the number of hands they shook. Prove that if we asked everyone who attended the party after the party was over how many hands they shook and added all of those numbers up, that we would get an even number.

5) The arithmetic mean of two numbers x and y is $\frac{x+y}{2}$ while their geometry mean is $\sqrt{xy}$. Prove that the arithmetic mean of x and y is at least as big as their geometric mean. (Hint: show that the difference of the two is non-negative by rewriting this difference as a perfect square. It will also help to analyze twice this difference and set a = $\sqrt{x}$ and b = $\sqrt{y}$.)