Recitation #1 Warm-Up Problems
1/10/2014

1) Consider the sequence 1, -2, 3, -4, … (More formally, the n^{th} term of the sequence is $(-1)^{n+1}n$.) What is the average of the first 200 terms of the sequence?

2) How many two digit positive integers, n, have the property that the sum of n and the number obtained by reversing the order of its digits is a perfect square? List these numbers.

3) The average of 10 positive integers (not necessarily distinct) is 57. If the smallest of these integers is 7, what is the largest possible integer that could be on the list?

4) With proof, determine all possible real values a such that the equation $x^2 + ax + a^2 = 0$ has two distinct real roots.

5) Find the two values of x for which the following equation is true: $\log_2(x^2 + 4x) = 5$. (Note: by definition, if $a^b = c$, then $\log_a c = b$.)

Recitation #1: Logic Problems
1/10/2014

1) Create a truth table for the following proposition. Include columns for intermediate parts.

$$(p \wedge \bar{q}) \to (r \vee \bar{p})$$

2) Below are the truth tables for NAND, NOR, and XOR (⊕). Write definitions for these operators in terms of AND, OR, and NOT. Try to explain in words what each one means.

A	B	A NAND B
0	0	1
0	1	1
1	0	1
1	1	0

A	B	A NOR B
0	0	1
0	1	0
1	0	0
1	1	0

A	B	A ⊕ B
0	0	0
0	1	1
1	0	1
1	1	0

3) The operator NAND, described above, can be called "computationally complete," or a "sole sufficient operator." That means that by itself, it can replace any of the other logical operators. How would you rewrite the statement "¬P" using only NAND?

4) Prove the following using the laws of logic and the implication identity:

$$(\ (p \vee \neg p)\ \wedge\ (\ q \vee\ \neg(\neg q \vee \neg r)\ )\ )\ \vee\ (\ (p \vee\ \neg p)\ \wedge\ \neg q) \leftrightarrow T$$

5) Prove the following using the laws of logic and the implication identity:

$$(\neg q \to \neg p) \to ((p \wedge q) \vee r)) \leftrightarrow r \vee p$$