Recitation #14 Warm-Up Problems
4/18/2014

1) Ed and Ann both have lemonade with their lunch. Ed orders the regular size. Ann gets the large lemonade, which is 50% more than the regular. After they consume ¾ of their drinks, Ann gives Ed a third of what she has left, and 2 additional ounces. When they finish their lemonades, they realize that they both drank the same amount. How many ounces of lemonade did they drink together?

2) For how many positive integers n is $\frac{n}{30-n}$ also a positive integer?

3) Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, *abc* miles were displayed on her odometer, where *abc* is a three-digit number with a ≥ 1 and $a + b + c \leq 7$. At the end of the trip, the odometer showed *cba* miles. What was the odometer reading when the trip was over?

4) A rectangular box has a total surface area of 94 square inches and the sum of the lengths of its edges is 48 inches. What is the sum of the lengths in inches of all of its interior diagonals?

5) Let P be a cubic polynomial with P(0) = k, P(1) = 2k and P(-1) = 3k. What is P(2) + P(-2)? (Note: It's impossible to determine P(2) or P(-2) since neither of these values is uniquely determined based on the given information. But, it turns out that P(2) + P(-2) can only take on one value, in terms of k.)

Recitation #14 Function Problems

1) Let $f(x) = x^2 - 4x + 7$, with a domain of $x \leq 2$. What is $f^{-1}(x)$? What is $f^{-1}(x)$'s domain and range?

2) Let $f(x) = 2^{3x-7}$ and $g(x) = 2x^2 + 5$. What are both $f(g(x))$ and $g(f(x))$? What is the minimum value of $f(g(x))$?

3) Let $f : A \rightarrow B$ and $g : B \rightarrow A$ be functions such that

(i) $\forall y \in B, (f \circ g)(y) = y$
(ii) $\exists x \in A : (g \circ f)(x) \neq x$

Prove that f is surjective (onto), but not injective (one-to-one).

4) Let $f : A \rightarrow B$ and $g: B \rightarrow A$ denote two functions. Suppose $g \circ f(x) = x$ for all $x \in A$. Answer the following two questions:

a) Prove that the function f is an injection. (**Hint:** Prove that if $f(x) = f(y)$, then $x = y$.)

b) Prove that the function g is a surjection. (**Hint:** Let $y \in A$, we need to find $x \in B$ such that $g(x) = y$. Try $x = f(y)$.)

5) Let $f(x) = 3x - 2$, for all $x < 3$
$= x + 4$, for all $x \geq 3$.

Prove or disprove: f(x) is a bijection over $R \rightarrow R$.