

Recitation #13 Warm-Up Problems
4/11/2014

- 1) The difference between a two-digit number and the number obtained by reversing its digits is 5 times the sum of the digits of either number. What is the sum of the two digit number and its reverse?
- 2) Five consecutive integers starting with a have an average of b . What is the average of 5 consecutive integers starting with b , in terms of a ?
- 3) David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way and arrives at the airport 30 minutes early. How many miles is the airport from his home?
- 4) A fancy bed and breakfast inn has 5 rooms, each with a distinctive color-coded décor. One day 5 friends arrive to spend the night. There are no other guests that night. The friends can room in any combination they wish, but with no more than 2 friends per room. In how many ways can the innkeeper assign the guests to the rooms? (Note: The total number of assignments without the maximum number of friends in a room restriction is 5^5 , because we are counting each person and each room as distinguishable. So, each unique room placement can be expressed as a ordered 5-tuple of rooms, where the i^{th} item in the tuple is the room the i^{th} person is staying in.)
- 5) The product $8 \times (8888888 \dots 888)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k ?

Recitation #13 Relations Problems

1) If A is the set $\{1, 2, 3, 4\}$ give an example of a relation on A that is:

- a) reflexive and symmetric, but not transitive.
- b) reflexive and transitive, but not symmetric.
- c) symmetric and transitive, but not reflexive.

2) Prove that the following relation R defined over $Z \times Z$ is transitive:

$$R = \{(x, y) \mid x + 3y = 4c, \text{ for some integer } c.\}$$

3) Prove that a relation $R \subseteq A \times A$ is transitive if and only if $R^2 \subseteq R$.

4) Let a set A contain 10 elements. How many relations over $A \times A$ are reflexive and symmetric?

5) Let $R = \{(a, b) \mid a, b \text{ and } b \text{ are positive integers that form the sides of a triangle}\}$ over the set Z^+ . Determine, with proof, whether or not R is (a) reflexive, (b) symmetric, and (c) transitive.