

Recitation #11 Warm-Up Problems
3/28/2014

- 1) For each positive integer n , the mean of the first n terms of a sequence is n . What is the 2008th term of the sequence?
- 2) Bricklayer Brenda takes 9 hours to build a chimney alone, and bricklayer Brandon takes 10 hours to build it alone. When they work together, they talk a lot and their output decreases by 10 bricks per hour, total. Working together, they build a chimney in 5 hours. How many bricks are in the chimney?
- 3) A rectangular floor measures a feet by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half of the area of the entire floor. List all the possible ordered pairs (a, b) .
- 4) Points A and B are on a circle with radius 5 and $AB = 6$. Point C is the midpoint of the minor arc AB. What is the length of the line segment AC?
- 5) A circle has a radius of $\log_{10}(a^2)$ and a circumference of $\log_{10}(b^4)$. What is $\log_a b$?

Recitation #11 Counting Problems

1) The dean of a certain college has a pool of 10 chemists, 7 psychologists, and 3 statisticians from which to form a committee.

a) The chemist, Professor C, and the psychologist, Professor P, do not get along. How many 5-member committees can be formed that do not have both Professor C and Professor P on them?

b) How many 5-member committees can be formed so that the number of chemists is greater than or equal to the number of psychologists and the number of psychologists is greater than or equal to the number of statisticians?

2) Johnny lives in New York City on a grid system. His apartment is at the corner of 25th Street and 8th Avenue while his work place is located at 42nd Street and 2nd Avenue. Streets and Avenues are present at every block.

a) If Johnny always walks either North (street numbers are increasing) or East (avenue numbers are decreasing), in how many ways can he walk to work?

b) If Johnny wants to get coffee at the intersection of 40th Street and 6th Avenue on his way to work, how many ways can he go?

c) If Johnny wants to avoid the intersection of 40th Street and 6th Avenue on his way to work, how many ways can he go?

d) If Johnny wants to avoid the intersection mentioned in part (c) AND also avoid the intersection of 33rd Street and 7th Avenue on his way to work, how many ways can he go?

3) How many non-negative solutions are there to the equation $a + b + c + d + e = 27$ with $a > 5$?

4) A farmer wants to plant 3 birch trees, 4 oak trees and 5 palm trees in a row such that no two palm trees are adjacent. How many ways can he do it?

5) How many integers between 1 and 1,000,000 have a sum of digits equal to 9? For example, 333 and 10,125 are valid, but 9,999 and 12,111,111 are invalid.