

COT 3100 Recitation #11: Final Exam Review
11/28/2016-12/2/2016

1) Using mathematical induction on n , prove for all positive integers n that

$$\sum_{i=1}^n \frac{1}{i(i+1)(i+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

2) There are three people, Adam(A), Belinda(B) and Celine(C), in a room. Each has a hat on and is holding a pet. The colors of the three hats are red(R), orange(O) and purple(P) and the pets are a dog(D), a guinea pig(G) and a hamster(H). Consider the following given information:

- 1) The person wearing the red hat is not holding the guinea pig.
- 2) Celine is not holding the hamster.
- 3) The person wearing the orange hat is holding the hamster.
- 4) Belinda is holding the dog.

Using any technique, determine which hats each of the individuals is wearing and which pets each of the individuals is holding.

Person	Hat Color	Pet
Adam		
Belinda		
Celine		

3) Prove or disprove the following assertion about finite sets A and B :

$$P(A) \cup P(B) \subseteq P(A \cup B)$$

Recall that $P(A)$ is simply the set of all subsets of A .

4) (15 pts) NTH (Number Theory)

This question will concern all positive integer solutions to the equation $a^2 + b^2 = c^2$ where $\gcd(a, b) = 1$, $\gcd(a, c) = 1$ and $\gcd(b, c) = 1$.

a)(5 pts) Let n be an even integer. Prove that $n^2 \equiv 0 \pmod{4}$.

b)(5 pts) Let n be an odd integer. Prove that $n^2 \equiv 1 \pmod{4}$.

c)(5 pts) Prove that for all solutions of $a^2 + b^2 = c^2$ where $\gcd(a, b) = 1$, $\gcd(a, c) = 1$ and $\gcd(b, c) = 1$, that c must be odd and that exactly 1 of a and b must be odd.

5) How many subsets of 5 cards out of 52 regular playing cards contain at least 1 card of each suit?

6) Each part will concern the situation of rolling three fair six-sided dice. Label these dice A, B and C. Note: Please simplify each of these answers as fractions in lowest terms.

a) (5 pts) What is the probability that the sum of the three dice is 5?

b) (5 pts) Given that die A shows a 3, what is the probability that the sum of dice A and B equals what is showing on die C?

c) (5 pts) Given that one of the three dice shows a 1 (note that we don't know which one of A, B and C), what is the probability that all three sum to three?

7) Let $f(x) = 3x^2 - 12x + 7$ with a domain of $x \in (-\infty, 2]$. Determine $f^{-1}(x)$ and its domain and range.

8) Recall that a bitwise and (&) between two integers is the result of taking the and of each pair of corresponding bits in the two operands. For example $11 \& 13$ is 9, because 11 is 1011 in binary, 13 is 1101 in binary, and both of these numbers have 1s in the first and last locations, thus their bitwise and is 1001, which is equal to 9. Define the following relation over **non-zero** 32-bit integers:

Let $R = \{ (a, b) \mid a \text{ and } b \text{ are 32-bit } \underline{\text{non-zero}} \text{ integers with } (a \& b) \text{ equal to } 0 \}$

Determine, with proof, whether or not R is (a) irreflexive, (b) symmetric and (c) transitive.