Recitation #10 Warm-Up Problems
3/21/2014

1) A teacher gave a test to a class in which 10% if the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each junior receive on the test?

2) A geometric series $a + ar + ar^2 + \ldots$ has a sum of 7, and the terms involving odd powers of r have a sum of 3. What is are the values of a and r?

3) If a is a nonzero integer and b is a positive number such that $ab^2 = \log_{10} b$, what is the median of the set {0, 1, a, b, 1/b}?

4) A traffic light repeatedly runs through the following cycle: 30 seconds green, 3 seconds yellow, 30 seconds red. Leah picks a random 3 second interval to watch the light. What is the probability that the color changes while she is watching?

5) Let a, b and c be digits with $a \neq 0$. The three-digit number abc lines one third of the way from the square of a positive integer to the square of the next larger integer. The integer acb lies two thirds of the way between the same two squares. What is $a + b + c$?

Recitation #10 Counting Problems

1) Using the multiplication principle, determine the number of passwords of length 8 we can make where each character must be an uppercase letter. Imagine relaxing the length restriction so that a password could be 8 or fewer uppercase characters (but can't be the empty string). Write a summation equal to the valid number of passwords in this situation. Using the formula for a finite geometric sequence, simplify the summation you derived. Using the work in your example, determine the number of passwords of length N or fewer characters (with at least one character) where each character is chosen from one of c choices.

2) Using the multiplication principle, determine the number of orderings of the numbers 1, 2, 3, and 4. Using similar thinking, generalize the result to determine the number of orderings of the numbers 1, 2, 3, …, n.

3) Using the multiplication principle, determine how many unique lists of length 3 you can make from the numbers 1, 2, 3, 4, 5, where you are allowed to use each number at most one time. Using similar thinking, generalize the result to determine the number of unique lists of length k you can make from the numbers 1, 2, 3, …, n. You may assume that $1 \leq k \leq n$. Express your answer in pi notation. See if you can come up with an alternate expression equal to the same answer using two factorials in a fraction, one in the numerator and one in the denominator.

4) If we look at all of the lists in question three, we see that some lists, such as 1, 4, 3 and 3, 1, 4, both contain the exact same three elements. We can group the lists in question three into groups such that each group contains lists of the exact same three elements. How many lists are in each group? How many different groups are there? Each of these groups represents one combination of 3 items out of 5. Using our generalized answer to #3 and our generalizing our answer to the question, "How many lists are in each group?", we can determine a general answer to the question, "How many combinations of k items are there out of n?", in terms of both n and k. Give your answer to this question. You may use factorials, the pi symbol, or both.

5) An ascending number is a number where each subsequent digit is strictly greater than the previous digit. For example, 237, 1389 and 45 are all ascending numbers. How many ascending numbers of length 5 are there?