Fall 2017 COT 3100 Recitation #7: Counting
10/30-11/3/2017

Warm-Up Problems

1) In a group of 50 blocks, each block has either a number of letter on it and is either red or blue. 14 blocks are blue and have numbers on them. 31 have letters on them and 18 are red. How many blocks are red with letters on them?

2) The number 2^{48} - 1 is exactly divisible by two numbers between 60 and 70. What are they?

3) The current in a river is flowing steadily at 3 miles per hour. A motor boat which ravels at a constant rate in still water goes downstream 4 miles and then returns to its starting point. The trip takes one hour, excluding the time spent in turning the boat around. What is the ratio of the downstream to the upstream rate?

4) A teen age boy wrote his own age after his father's. (Both are in between 10 and 99, inclusive.) From this new 4 digit number he subtracted the absolute value of the difference of their ages to get 4289. What is the sum of their ages?

5) Given the geometric progression $10^{1/11}$, $10^{2/11}$, $10^{3/11}$, etc. What is the least positive integer n such that the product of the first n terms exceeds one billion (1,000,000,000)?

Counting Questions

6) Define a peak number to be a seven digit number, with unique digits such that the first four digits are in ascending order and the last four digits are in descending order. For example, 1357642 is a peak number. Note that a peak number may NOT start with the digit zero. How many peak numbers are there?

7) How many permutations of SEPTEMBER have all three Es appearing consecutively?

8) A car dealership owner is ordering exactly 100 cars from headquarters. In particular, she is ordering the following cars: Civic, Accord, Element, Odyssey, Pilot, Ridgeline and Fit. She must order at least one of each car, but order no more than 20 Civics. (Some people are concerned that Civics will take over the world and our fearless dealership owner wants to address these concerns!) In how many ways can she place her order?

9) In a gumball machine there are 32 red gumballs, 14 green gumballs, 30 white gumballs, and 5 purple gumballs. A devoted customer purchases 10 gumballs. How many combinations of gumballs can the customer receive? (Remember the order in which you receive the gumballs does not matter. Only the total set of 10 gumballs matters.)

10) How many integers in between 1 and 10^7 are divisible by 2, 5 or 7?