COT 3100 Recitation #10: Exam #3 Review - Counting, Probability
11/14-11/18/2016

1) How many 7 digit phone numbers do not contain any 0s and do not have any repeated digits?

2) A department has 10 full professors, 5 associate professors and 8 assistant professors. A committee of four professors is to be formed with at least one professor from each rank. How many distinct committees can be formed under these restrictions?

3) Jamal's high school offers 10 AP courses. He plans to take 3 AP courses his junior year and 4 AP courses his senior year. How many different schedules of AP courses can he put together for his junior and senior years? (Note: We count two schedules as different if one contains a course the other doesn't or if one has a course scheduled for the junior year while the other has the same course scheduled for the senior year.)

4) How many 15 digit numbers (no leading 0s) have each of their digits in sorted order from left to right? (For example, the number 111122355679999 should be counted.)

5) How many diagonals does a convex polygon of n sides have? (A diagonal is a line segment connecting two vertices of the polygon that resides strictly *inside* the polygon, except for the two vertices themselves.)

6) Using the algebraic method prove that $\binom{n+1}{k} = \frac{n+1}{k}\binom{n}{k-1}$.

7) An urn contains 10 red balls and 5 black balls. Three balls are drawn from the urn without replacement. What is the probability that exactly 2 of the 3 balls are red?

8) Given that Amir has studied, his chance of getting an A on an exam is 70%. If he does not study, his chance of getting an A on an exam is 25%. Amir studies before 40% of his exams. Given that Amir got an A on his first biology exam, what is the probability that he studied for it?

9) Sarita will win a prize if she gets a hole in one at the mini-golf place. Her chance of getting a hole in one at each hole is 10% and there are 9 holes throughout the whole course. What is the probability that she'll get the prize?

10) An unfair coin lands heads 75% of the time. If the coin is flipped 6 times, what is the chance that it will land heads exactly 4 times?

11) In a new casino game "Dice Wars", a contestant rolls a pair of fair 6 sided dice. If they total an even number, they stop rolling. If the total an odd sum on the first roll, they will continue rolling the pair of dice until they get their original total. You may assume that if the probability of success on an individual trial is p, then the expected number of trials one must perform to achieve their first success is $\frac{1}{p}$. What is the expected number of times a contestant will roll the dice in Dice Wars?

12) When a contestant plays Dice Wars, they wager $10 for each time they play. If they get an even number on their first roll, they receive $5, so in effect, they've lost only $5. Alternatively, if they roll an odd sum on their first roll, they only win any money back if they roll that exact same total on their second roll. In these cases, they get back $10 times the value of the roll. So if you roll a 7 followed by another 7, the contestant would win $70, for a net gain of $60. (Note: The contestant continues rolling even if they don't win due to other side bets that observers can make.) What is a contestant's expected net outcome for playing the game once? (For example, if their expected winnings are $8 for playing once, their expected net outcome per game is losing $2.)