Fall 2017 COT 3100 Recitation #6: Counting
10/23-10/27/2017

Warm-Up Problems

1) What is the fourth power of $\sqrt{1 + \sqrt{1 + \sqrt{1}}}$?

2) The sum of an infinite geometric series with common ratio r such that $| \, r \, | < 1$ is 15, and the sum of the squares of the terms of this series is 45. What is the first term of the series?

3) The number 10!, when written in base 12, ends in how many zeroes? (Challenge: figure out a fast algorithm to solve this problem when the factorial can be quite large and the base is anything that is easy to prime factorize.)

4) It is now in between 10:00 and 11:00 o'clock, and six minutes from now, the minute hand of the watch will be exactly opposite the place where the hour hand was three minutes ago. What is the exact time now?

5) Find the sum of the digits in all of the integers from 1 to 10000, inclusive.

Counting Questions

6) PERSON ID numbers adhere to the following rules:

    (i) A string of 8 digits
    (ii) The leading digit can not be 0.
    (iii) The second digit can not be the same as the first digit.
    (iv) The substring "90009" can NOT appear in the number.

How many valid PERSON ID numbers are there?

7) The U. S. Senate has 21 women and 79 men. How many committees of 5 senators with at least 2 women can be formed?

8) How many permutations of ENGINEERING do NOT contain consecutive vowels?

9) A "vowel-ascending" string is one where each of the vowels (AEIOU) within the string appear in alphabetical order. For example "BARENECESSIT" is a vowel-ascending string since all of the As come before all of the Es which come before all of the Is. How many vowel-ascending strings can be formed with the letters in the word ENGINEERING?

10) Bob is an ant walking on the Cartesian plane, starting at (0, 0) and ending at (12, 10). He always walks a single unit either in the positive x-axis direction or positive y-axis direction and arrives at his destination after walking 22 units. If he isn't allowed to go to (8, 3), in how many different ways can he make his journey?