Fall 2017 COT 3100 Recitation #5: Exam 2 Review
10/16-10/20/2017

Warm-Up Problems

1) If the convex area bounded by the x-axis, the line y=mx+4, x = 1 and x = 4 is 7, what is the value of m?

2) The difference in the areas of two similar triangles is 18 square feet, and the ratio of the larger area to the smaller area is the square of an integer. The area of the smaller triangle, in square feet, is an integer and one of its sides is 3 feet. What is the length of the corresponding side on the larger triangle?

3) A dealer bought n radios for a total of d dollars, where d is a positive integr. He contributed 2 radios to a community bazaar at half their cost. The rest he sold at a profit of $8 for each radio sold. If his overall profit was $72, what is the smallest possible value of n?

4) How many ordered pairs of positive integers (x, y) satisfy the equation 3x + 5y = 501?

5) Two candles of the same length are made of different materials so that one burns out completely at a uniform rate in 3 hours and the other in 4 hours. At what time should both of the candles be lit so that at 4 pm, one stub is twice the length of the other?

Exam Review Problems - Number Theory, Induction

6) Find all integer solutions (x, y) for the equation 14x + 19y = 13.

7) What is the smallest integer that has precisely 39 divisors and only two unique prime factors? (Note: Just provide the prime factorization of this integer as it's reasonably large.)

8) Calculate the following sum for a few small values of n and conjecture a guess for the value of the summation, in terms of n. Then, prove your guess via mathematical induction:

$$\sum_{i=1}^{n} \frac{1}{i(i+1)}$$

9) Use mathematical induction on n to prove for all positive integers n, x - 1 is a factor of $x^n - 1$.

10) Prove for all non-negative integers n that $3 \mid (n^3 + 2n)$.