

COT 3100 Recitation #8: Counting
10/24-28/2016

Warm-Up Problems

- 1) Tom's age is T years, which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is T/N ?
- 2) The geometric series $a + ar + ar^2 + \dots$ has a sum of 7, and the terms involving odd powers of r have a sum of 3. What is $a + r$?
- 3) A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each junior receive on the test?
- 4) If a is a non-zero integer and b is a positive integer such that $ab^2 = \log_{10} b$, what is the median of the set $\{0, 1, a, b, 1/b\}$?
- 5) The first 2007 positive integers are each written in base 3. How many of these base-3 representations are palindromes? (A palindrome is a number whose digits read the same forwards and backwards. For example, 11011 and 1221 are both palindromes in base 3.)

Counting Problems

- 6) How many permutations (of length 11) are there of the letters in the word MATHEMATICS?
- 7) A string, $c_1c_2c_3\dots c_n$, is called an ascending string if c_i comes strictly before c_{i+1} for all $1 \leq i \leq n-1$. How many ascending strings of length 4 can be formed?
- 8) How many integers in between 1 and 1000 are divisible by 2, 3 or 5?
- 9) How many permutations of "ATTTCCCAAGGG" are there such that all "C"s and "G"s appear together consecutively?
- 10) How many permutations of 4 As, 5 Bs and 6 Cs do not contain two consecutive CC's in a row?