Warm-Up Problems

1) How may real values of x satisfy the following equation: $(2^{6x+3})(4^{3x+6}) = 8^{4x+5}$?

2) In a given arithmetic sequence the first term is 2, the last term is 29 and the sum of the terms is 155. What is the common difference of the sequence?

3) If $f(n+1) = \frac{2f(n)+1}{2}$, and $f(1) = 2$, what is $f(101)$?

4) At his usual rowing rate, a man rows downstream 15 miles in 5 fewer hours than it takes him to return. If he doubles his usual rowing rate, the time downstream is only 1 hour less than the time upstream. What is the rate of the stream, in miles per hour?

5) Three men, Bob, John and Steven, working together do a job in 6 hours less time than Bob alone, in 1 hour less time than John alone, and in one half the time needed by Steven, when working alone. How many hours would Bob and John working together take to do the job?

Induction Problems

6) The nth Harmonic number, denoted H_n is defined as follows: $H_n = \sum_{i=1}^{n} \frac{1}{i}$

Prove that the following equation is true for all positive integers n, using induction on n:

$$\sum_{i=1}^{n} \frac{i}{i+1} = (n+1) - H_{n+1}$$

7) Use induction on n to prove the following inequality for all positive integers n:

$$\sum_{i=0}^{n} 3^i < \frac{3^{n+1}}{2}$$

8) The Fibonacci numbers are defined as follows: $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$, for all integers n>1. Prove the following formula for all positive integers n:

$$\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$$