COT 3100 Recitation #7: Exam #2 Review - Induction
10/17-10/21/2016

1) Use induction on n to prove that $4^{2n} - 15n - 1$ is divisible by 225 for all non-negative integers n.

2) Using induction on n, prove that the following formula is true for all positive integers n.

$$\sum_{i=1}^{n} \frac{i(i+1)(i+2)}{6} = \frac{n(n+1)(n+2)(n+3)}{24}$$

3) Use induction to show that $\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ -2^n + 1 & 2^n \end{pmatrix}$ for all positive integers n.

4) The n^th^ Harmonic number, denoted H_n is defined as follows:

$$H_n = \sum_{i=1}^{n} \frac{1}{i}$$

Prove that the following equation is true for all positive integers n, using induction on n:

$$\sum_{i=1}^{n} \frac{i}{i+1} = (n+1) - H_{n+1}$$

5) Use induction on n to prove the following inequality for all positive integers n:

$$\sum_{i=0}^{n} 3^i < \frac{3^{n+1}}{2}$$

6) The Fibonacci numbers are defined as follows: $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$, for all integers n > 1. Prove the following formula for all positive integers n:

$$\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$$