

Fall 2017 COT 3100 Recitation #3: Number Theory Practice
9/25-9/29/2017

Warm-Up Problems

- 1) What is the smallest possible positive integer x such that $1260x = N^3$, where N is some positive integer?
- 2) A list of 50 numbers has an average of 38. If we remove the values 45 and 55 from the list, what is the new average of the remaining 48 numbers?
- 3) a , b and c form an arithmetic sequence with a non-zero common difference. If a is increased by 1 and c is increased by 2, the resulting sequence is a geometric progression. What is b ?
- 4) A circle is inscribed in an equilateral triangle. A square is inscribed in that circle. What is the ratio of the area of the equilateral triangle to the square?
- 5) For what values of k do the equations $y = x^2$ and $y = 3x + k$ have two identical solutions?

Number Theory Problems

- 6) Find the quotient and remainder for the following division problems:

i) $a = 147$, $b = 6$

ii) $a = 12345$, $b = 106$

iii) $a = 77$, $b = 96$

iv) $a = 421$, $b = 77$

v) $a = 1776$, $b = 148$

- 7) Without the aid of a calculator, find the value of the remainders for each of the following divisions:

i) $a = 47^4$, $b = 15$

ii) $a = 10106^4$, $b = 10$

iii) $a = 77^{10}$, $b = 40$

iv) $a = 750^9$, $b = 47$

- 8) Convert each of the following values to base 10:

i) $2A6B_{16}$

ii) 2376_8

iii) 44341_5

iv) 11011010111_2

- 9) Prove the following assertion: If n is a composite integer, then there exists a divisor a of n such that $1 < a \leq \sqrt{n}$. Determine when the only divisor of a composite integer n is precisely \sqrt{n} .

- 10) In base r , $1000 - 340$ equals 440 . What is r ?