COT 3100 Recitation #6: Sums and Matrix Practice
10/10-14/2016

Warm-Up Problems

1) Mary is 20% older than Sally and Sally is 40% younger than Danielle. The sum of their ages is 23.2 years. How old will Mary be on her next birthday?

2) For how many real values of x is $\sqrt{120 - \sqrt{x}}$ an integer?

3) Two farmers agree that pigs are worth $300 and that goats are worth $210. When one farmer owes the other money, he pays the debt in pigs and goats, with "change" received in the form of goats or pigs, as necessary. What is the amount of the smallest positive debt that can be resolved this way?

4) The function f has the property that for each real number x in its domain, 1/x is also in its domain and $f(x) + f\left(\frac{1}{x}\right) = x$. What's the largest set of real numbers that can be the domain of f?

5) Let A, B and C be the centers of three mutually tangent circles such that the distance between A and B is 3, the distance between A and C is 4 and the distance between B and C is 5. What is the sum of the areas of the three circles? (Hint: the sum of the radii of the circles with centers A and B is 3, and a similar deduction is true about the sum of the radii of the other two pairs of circles.)

Summation and Matrix Problems

6) Determine $\sum_{i=1}^{2n}(3i + 5)$, in terms of n.

7) Determine $\sum_{i=n+1}^{2n}((2i + 1)(i - 3))$.

8) Consider an arithmetic sequence with the first term 22 and the tenth term 67. What is the sum of the first 20 terms of the sequence?

9) Calculate the matrix product $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}^2$.

10) Calculate the following Boolean product $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$.