COT 3100 Recitation #5: Number Theory Practice
9/26-30/2016

Warm-Up Problems

1) The average of 20 numbers is 30 and the average of 30 other numbers is 20. What is the average of all 50 numbers?

2) Selena and Trina live 13 miles apart. Yesterday Selena started to ride her bicycle toward Trina's house. A little later Trina started to ride her bicycle towards Selena's house. When they met, Selena had ridden for twice th length of time as Trina and at four-fifths of Mike's rate. How many miles had Trina ridden when they met?

3) Let A, M and C be digits with

$$(100A + 10M + C)(A + M + C) = 2005$$

What is A?

4) A wooden cube n units on a side is painted red on all six faces and then cut into n^3 unit cubes. Exactly one-fourth of the total number of faces of the unit cubes are red. What is n?

5) A line passes through A(1, 1) and B(100, 1000). How may other points with inteer coordinates are on the line and strictly in between A and B?

Number Theory Problems

6) Find all integer solutions (x, y) to the equation 35x + 49y = 42.

7) Find all integer solutions (x, y) to the equation 91x + 130y = 47.

8) Find all integer solutions (x, y) to the equation 45x + 91y = 6.

9) Determine 47^{-1} mod 111. Using this result, solve for x in the equation $47x \equiv 71 \ (mod\ 111)$.

10) Utilize the fast modular exponentiation algorithm shown in the text (and in class) to find an integer m, $0 \leq m < 29$ such that $2^{27} \equiv m \ (mod\ 29)$.