

COT 3100 Recitation #4: Number Theory Practice
9/19-23/2016

Warm-Up Problems

- 1) For how many ordered pairs of positive integers (x, y) is $3x + 5y = 2016$?
- 2) A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?
- 3) The sum of 49 consecutive integers is 7^5 . The median of these 49 consecutive integers is 7^k for some integer k . What is k ?
- 4) The average value of all the pennies, nickels, dimes and quarters in Paula's purse is 20 cents. If she had one more quarter, the average value would be 21 cents. How many dimes does she have in her purse?
- 5) A sequence of three real numbers forms an arithmetic progression with the first term 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression.

Number Theory Problems

- 6) Use the Euclidean Algorithm to find the greatest common divisor of 565 and 235.
- 7) For positive integers, m , find all integers x with $0 \leq x < 5$ such that

$$m^2 - m + 11 \equiv x \pmod{5}$$

(Hint: If we plug in two different values of m that are equivalent mod 5, we will obtain the same result for x . Thus, it's enough to specifically plug in $m = 0, 1, 2, 3$ and 4 to obtain all possible results for x .)

Based on your list, is it possible for $m^2 - m + 11$ to be divisible by 5 if m is a positive integer?

- 8) Using the division algorithm repeatedly, convert 1431 in base 10 to base 7.
- 9) Use the textbook's modular exponentiation algorithm to calculate the remainder when 2^{27} is divided by 11.
- 10) Use the Sieve of Eratosthenes to generate a list of all prime numbers less than 50.