COT 3100 Recitation #3: Exam #1 Review

The questions below are the questions from last fall's Exam #1.

1) (8 pts) Complete filling out the truth table below that evaluates the logical expression. For ease of reading, **<u>please use 0 for false and 1 for true.</u>**

$$(p \vee \bar{q}) \rightarrow (\bar{r} \rightarrow \bar{q}).$$

p	q	r	$p \vee \bar{q}$	$\bar{r} \rightarrow \bar{q}$	$(p \vee \bar{q}) \rightarrow (\bar{r} \rightarrow \bar{q})$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

2) (8 pts) Using the laws of logic, show that two following expressions are logically equivalent.

$$(p \wedge (p \vee \bar{r} \vee q)) \vee ((q \wedge r) \vee (q \wedge \bar{r})) \qquad \bar{p} \rightarrow q$$

3) (8 pts) Use the rules of inference to make the following argument:

$s \rightarrow (p \rightarrow r)$
$s \vee t$
$\bar{t}$
$p \vee q$

$q \vee r$

4) (5 pts) For an open statement P(x, y), it is known that $\exists x[\forall y(P(x,y)]$. Is it necessarily true that $\forall y[\exists x(P(x,y)]$? If it is the case, prove it, if the assertion is false, create a specific open statement P(x, y) for which $\exists x[\forall y(P(x,y)]$ is true and $\forall y[\exists x(P(x,y)]$ is false and explain why the first is true and the second is false.

5) (12 pts) Prove or disprove: If n is an integer such that $n \equiv 1\ (mod 6)$, then $n^2 \equiv 1\ (mod\ 24)$. (Note: You may use the result that we previously proved in homework, namely, for any integer a, $a(3a+1)$ is an even integer.

6) (10 pts) Prove or disprove the following assertion about finite sets A and B, taken from the positive integers (Note: $\wp$ indicates Power Set.):

$$\wp(A) - \wp(B) \subseteq \wp(A - B).$$

7) (10 pts) Prove or disprove the following assertion about finite sets A, B, and C taken from the positive integers:

$$(A - C) - (B - C) \subseteq (A - B)$$

8) (10 pts) Consider finite sets A, B and C where we know the cardinalities of the following sets:

$$|A \cap C| = 0$$
$$|A \cup B| = 20$$
$$|B \cap C| = 5$$
$$|A \cup B \cup C| = 22$$

Determine |C|. (Note: Solutions that only use a Venn-Diagram to determine the answer will receive a maximum of 5/10 points. Only solutions that use a formal proof involving the Inclusion-Exclusion principle will earn full credit.)

9) (3 pts) Let $S = \{2x^2 + x - 6 = 0 | x \in Z\}$. (Note: Recall that Z is the set of integers.) Explicitly list each element that belongs to the set S. Put a circle around your final answer.

Note: Our exam will be at least one question based on the warm-ups and/or "extra" problems I've done in lecture.