Recitation #2 Warm-Up Problems

1) How many positive integers b have the property that $\log_b 729$ is a positive integer?

2) Two non-zero real numbers, a and b, satisfy ab = a – b. What are all the possible values of $\frac{a}{b} + \frac{b}{a} - ab$?

3) Let A, M and C be non-negative integers such that A + M + C = 12. What is the maximum value of AMC + AM + MC + AC?

4) Let f be a function for which $f\left(\frac{x}{3}\right) = x^2 + x + 1$. $f(3z) = ax^2 + bz + c$. Determine the values of a, b and c.

5) A checkerboard of 13 rows and 17 columns has a number written in each square, beginning in the upper left corner, so that the first row is numbered 1, 2, 3, …, 17, the second row 18, 19, 20, .., 24, and so on down the board. If the board is renumbered so that the left column, top to bottom, is 1, 2, 3, …, 13, the second column is 14, 15, 16, …, 26 and so on across the board, some squares have the same number in both numbering systems. What is the sum of these squares?

Recitation #2: Set Problems

Fill in the blanks for the following groups of statements. As for "showing your work," you should be able to explain in words what the notation in each question means and why your answer is right.

1) | { ∅, {0}, {1}, {11}, {0, 1}, {1, 2, 1 + 2} } | = ___________________

2) Let A = {0, 2, 4, 6, B = {0, 1, 2} and C = $\mathbf{Z}^+$. What is (A ∪ B) – C = _____________?

3) Given that A = {1, 3, 4, 5, 8}, A ∪ B = { 1, 2, 3, 4, 5, 6, 8} and A – B = {3, 5, 8}, what is B?

4) Prove or disprove the following statement about sets: A ∩ (B ∪ C) = (A ∩ B) ∪ C

5) Prove or disprove the following statement about finite sets A and B: If | A ∩ B | = | A ∪ B |, then A = B.

6) Using set laws, prove the following: $(C - (A \cup B)) \cup (B \cap C) \cup (A \cap C) = C$.

7) Prove or disprove for arbitrary sets A, B and C:

If $C \subseteq B$, then, $(A - B) \cup (B - C) = \neg C \cap (A \cup B)$.

8) Prove the following for arbitrary sets A, B and C, using proof by contradiction:

If (A - B) - C = A - (B - C) then A ∩ C = ∅.