Fall 2017 COT 3100 Recitation #1: Logic Practice
8/28-9/1/2017

Warm-Up Problems

1) The sum of Bob and Carol's age now is 100. Sixteen years ago, Bob was three times as old as Carol. How old was Bob when Carol was born?

2) The sequence a_1, a_2, …, a_n, $n > 1$ is an arithmetic sequence with $a_1 = 10$, $a_n = 50$, and a common difference, d, that is a positive integer. What is the sum of all the possible values of d?

3) How many positive integers less than 200 have an odd number of positive integer divisors?

4) Solve the following system of equations (find the only values for a, b, c and d that satisfy the following four equations):

$$a + b + c = 180$$
$$a + b + d = 197$$
$$a + c + d = 208$$
$$b + c + d = 222$$

5) If $\log(xy^4) = 13$ and $\log(x^3y^2) = 9$ what is $\log(xy)$?

Logic Problems

6) Use a truth table to prove the following logical equivalence of the two following Boolean expressions:

(a) $p \to (q \vee r)$ (b) $\bar{r} \to (p \to q)$

7) Use the Laws of Logic to show the equivalence of the two Boolean expressions from question 6.

8) Using the following premises:

$(\bar{p} \vee \bar{q}) \to (r \wedge s)$
$r \to t$
$\bar{t}$

Derive the conclusion p.

9) In class, Modus Ponens was proved using just the laws of logic. Prove the Rule of Resolution in the same manner. Namely, show that the following is a tautology via the laws of logic.

$$((p \vee q) \wedge (\bar{p} \vee r)) \to (q \vee r)$$

10) Find your own open numerical statement (with the universe of positive integers), $P(x, y)$ and $Q(x)$ such that exactly one of $\forall x(\exists y|P(x, y))$ and $\exists x(\forall y|P(x, y))$ is true.