COT 3100 Recitation #2: Logic Practice
8/29-9/2/2016

Warm-Up Problems

1) Al gets the disease algebritis and must take one green pill and one pink pill each day for two weeks. A green pill costs $1 more than a pink pill and Al's pills cost a total of $546 for the two weeks. How must does one green pill cost?

2) The second and fourth terms of a geometric sequence are 2 and 6, respectively. What are the possible values of the first term in the sequence?

3) Let s(x) denote the sum of the digits of the positive integer x. For example s(8) = 8 and s(123) = 1 + 2 + 3 = 6. For how many two-digit values of x is s(s(x)) = 3?

4) Cassandra sets her watch to the correct time at noon. At the actual time of 1:00 PM, she notices that her watch reads 12:57 and 36 seconds. Assuming that her watch loses time at a constant rate, what will be the actual time when her watch first reads 10:00 PM?

5) If $\log(xy^3) = 1$ and $\log(x^2y) = 1$ what is $\log(xy)$?

Logic Problems

6) Using the following premises:

$(p \wedge t) \rightarrow (r \vee s)$
$q \rightarrow (u \wedge t)$
$u \rightarrow p$
$\bar{s}$

Derive the conclusion $q \rightarrow r$.

7) In class, Modus Ponens was proved using just the laws of logic. Prove Modus Tollens in the same manner.

8) Find your own open numerical statement (with the universe of positive integers), $P(x, y)$ and $Q(x)$ such that exactly one of $\forall x(\exists y | P(x, y))$ and $\exists x(\forall y | P(x, y))$ is true.

9) In the course text (and later in class), a proof was shown indicating that $\sqrt{2}$ is an irrational number. Explain why "repeating" the proof for the $\sqrt{3}$ succeeds but trying it for $\sqrt{9}$ fails.

10) Prove or disprove the following statements over the universe of real numbers:

a) $\exists x \forall y(xy = y)$ b) $\forall x \exists y(3y^2 - 2y + 6 = x)$ c) $\forall x \forall y \exists z((z > x) \wedge (z < y))$