COT 3100 Recitation: Sum, Matrix, Recurrence Relation Practice

Warm-Up Problems

1) The 36 distinct integers from -15 to 20, inclusive fill a 6 x 6 grid such that each row and column sum to the same value. What is this row/column sum?

2) What integer less than 100 has the most number of divisors? If there is a tie, which is the smallest such number?

3) The r and s be the roots of the quadratic equation $x^2 + 3x - 6 = 0$. What is the quadratic equation with leading coefficient 1 that has roots $\frac{r}{s}$ and $\frac{s}{r}$?

4) Both Seema and Madison choose a positive integer at random, such that the probability they choose the integer k is 2^{-k}. What is the probability that Seema's number is smaller than Madison's?

5) Let S be the set of all positive integer divisors of 10^5. How many numbers are the product of two distinct elements of S?

Sum, Matrix, Recurrence Relation Problems

6) An arithmetic sequence $a_1, a_2, a_3, \ldots$ is such that $a_{10} = 40$ and $a_{25} = 10$. Determine both a_1 and the sum of the first 50 terms of the sequence.

7) An infinite geometric sequence $a_1, a_2, a_3, \ldots$ has a sum of 10. If the odd indexed terms (a_1, a_3, etc.) have a sum of 7, what is the common ratio of the sequence? What is the first term of the sequence?

8) Determine the following infinite summation: $\sum_{i=1}^{\infty}(2i-1)(\frac{1}{2})^i$.

9) Determine the following matrix product, $\begin{bmatrix} 2^n & 1-2^n \\ 3 & 2^{n-1} \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$, in terms of n.

10) The Fibonacci numbers are defined as follows:

$F_1 = 1$, $F_2 = 1$, for all $n > 2$, $F_n = F_{n-1} + F_{n-2}$.

For all $k > 2$, prove the following:

$$F_k^2 - F_{k-1}F_{k+1} = F_{k-2}F_k - F_{k-1}^2$$