COT 3100 Final Exam - Part A (Recitation Topics) - 25 pts (4/25/2024)

Last Name: ___________________ , First Name: _____________________

1) (5 pts) Steve and Alia must work together to staple 10,000 packets for a conference. Steve can staple one packet in 5 seconds. When working together, the two of them complete the job in five hours and 12 ½ minutes. How many seconds does it take Alia to staple a single packet?

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2) (5 pts) There exist positive integers x and y such that $x < y$ and $61 - xy = 5x + 6y$. Without trial and error, determine the values of x and y. Show your work to prove that you avoided trial and error.

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3) (5 pts) X and Y are both infinite geometric series with a sum of S. The first term of the series X is 10 and the first term of the series Y is 20. If the second term of the series X is 6, find the two following pieces of information: (a) S, (b) The common ratio of the series Y.

S = ________________ common ratio of series Y = ____________________

4) (5 pts) Let r and s be the roots of the quadratic equation $x^2 - 8x + 11 = 0$. Determine the quadratic equation with leading coefficient 1 which has the roots r^2 and s^2.

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5) (5 pts) Find the values of x and y which satisfy the following system of equations:

$$log_2 x + log_4(8y) = 10$$

$$log_4(4x) + log_2(2y) = 10$$

x = _______ y = _______

Scratch Page – Please clearly mark any work on this page you would like graded.

COT 3100 Final Exam - Part B - 100 pts (4/25/2024)

Last Name: __________________ , First Name: ____________________

1) (4 pts) Fill out the following truth table.

p	q	r	$p \rightarrow q$	$r \rightarrow p$	$(p \rightarrow q) \rightarrow (r \rightarrow p)$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

2) (8 pts) For each of the two statements over the universe of real numbers, determine if they are true or false, and provide justification for reason. (Your answer is worth 1 pt for each part and the justification is worth 3 points.) Circle

(a) $\forall x \exists y [y^2 = 2xy - x^2]$ Is it true? Yes No

Justification

(b) $\exists x \forall y [y^2 = 2xy - x^2]$ Is it true? Yes No

Justification

3) (10 pts) Let A = {1, 2} and B = {3, 4}. Answer the following questions about the set $\wp(A \times B)$.

(a) How many elements are in the set? ________

(b) What is the size of the largest element of $\wp(A \times B)$? ________

(c) List each element of $\wp(A \times B)$ that contains exactly 2 elements. (Note: more blanks than necessary are provided to obfuscate how many elements satisfy the requirement.)

_____________ , _____________ , _____________ , _____________ , _____________ ,

_____________ , _____________ , _____________ , _____________ , _____________

4) (6 pts) Let x and y be integers such that 13 | (2x + 5y). Prove that 13 | (73x – 19y).

5) (10 pts) Find all ordered pairs of integers (x, y) that satisfy the equation $294x + 128y = 10$.

6) (4 pts) How many unique primes appear in the prime factorization of 50!?

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7) (8 pts) A class has 15 boys and 18 girls. A team of 8 students, 4 boys and 4 girls must be selected. In how many ways can the team be selected? **Express your answer in prime factorized form.** (Note: 4 pts will be awarded for the symbolic answer and 4 pts will be awarded for the work on paper to simplify that to prime factorized form.)

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8) (10 pts) Using induction on n, prove, for all positive integers n, that

$$\sum_{i=1}^{n} \frac{1}{i(i+1)(i+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

9) (10 pts) Let a continuous random variable be defined as follows:

$$f(x) = c(x-1)^2, for\ 0 \leq x \leq 4$$
$$f(x) = 0, otherwise$$

(a) (3 pts) Determine the value of c.

(b) (4 pts) What is the expected value of the continuous random variable described above?

(c) (3 pts) What is the probability that x is in between 2 and 3 for this continuous random variable?

10) (10 pts) The Griswold family is buying some vacation souvenir t-shirts. They've agreed to by exactly 18 t-shirts total out of four different types of t-shirts, each representing a different tour the family took. Let the t-shirt types be A, B, C and D. The tourist shop only has 3 copies of t-shirt B and 7 copies of t-shirt D left. The Griswold children (there are 2 of them)) each insist on having at least one copy of t-shirt type A, thus at least 2 copies of t-shirt A must be purchased. In how many ways can the family buy their shirts while adhering to these requirements? (Leave your answer in factorials, combinations, etc.)

11) (10 pts) Let R be a relation defined over the universe of **non-empty sets of positive integers**, as follows:

$$R = \{(A, B) | A \cap B \neq \emptyset\}$$

With proof, determine if R is (a) reflexive, (b) irreflexive, (c) symmetric, (d) anti-symmetric, and (e) transitive.

12) (9 pts) Let $f(x) = \frac{2}{x+3} - 4$, with a domain of all real x except for x = -3.

(a) Determine $f^{-1}(x)$. Please express your answer in the form: $-(\frac{ax+b}{cx+d})$, where a, b, c and d are all positive integers.

(b) What is the domain for $f^{-1}(x)$?

(c) What is the range for $f^{-1}(x)$?

13) (1 pt) What geometric shape are the Pyramids of Giza? ____________________

Scratch Page – Please clearly mark any work you would like graded.