<u>Spring 2017 COT 3100 Final Exam: Part A (for all students)</u>

Last Name: ___________________ , First Name : _____________________

Date: April 27, 2017

1) (20 pts - Induction) Using induction on n, prove for all positive integers n that

$$\sum_{i=1}^{2n} (-1)^i i^3 = n^2(4n + 3)$$

2) (15 pts - Number Theory) A primitive Pythagorean Triple is an ordered triplet (a, b, c) of positive integers such that

$$a^2 + b^2 = c^2$$

with gcd(a, b, c) = 1. (For example, (3, 4, 5) is a Pythagorean triple, but (9, 12, 15) isn't because all three value share a common factor of 3 and (7, 13, 15) isn't either since $7^2 + 13^2 \neq 15^2$.)

In this question you'll prove that there are no primitive Pythagorean Triples with $c \equiv 0 \ (mod\ 3)$.

a) (6 pts) Let a be a positive integer not divisible by 3. Prove that $a^2 \equiv 1 \ (mod\ 3)$.

b) (4 pts) Let a and b be positive integers, at least one of which isn't divisible by 3. Find all possible remainders when $a^2 + b^2$is divided by 3.

c) (5 pts) Using results (a) and (b), prove that there are no primitive Pythagorean Triples with $c \equiv 0 \ (mod\ 3)$.

3) (20 pts - Probability) Alice and Bob play a game where they try to hit a target with an arrow. On any individual shot, Alice has a probability of p ($0 < p < 1$) of hitting the target while Bob has a probability of q ($0 < q < 1$) of hitting the target. The game proceeds as follows: Alice goes first and takes a shot. If she hits the target, she wins. If she doesn't, Bob gets a turn. On his turn, Bob takes a shot. If he hits the target, he wins. If he doesn't, it goes back to Alice's turn. They alternate until the target gets hit. The winner is whoever hits the target. Answer the following questions in relation to this game. Answer all questions in terms of p and q. **<u>(Put a box around your answers.)</u>**

a) (3 pts) What is the probability that the game lasts for 3 or more turns? (A single turn is one shot by one player. If Alice shoots and misses and Bob shoots and misses, they have completed 2 turns.)

b) (5 pts) Given that the game lasted fewer than 3 turns, what is the probability that Alice won?

c) (7 pts) What is the probability that Alice wins the game?

d) (3 pts) If Bob were to go first, what would Alice's probability of winning be? (Yes, it would change and her chance of winning will go down.)

e) (2 pts) Given that the original version of the game with Alice going first lasts at least 3 turns, what is the probability that Alice won? (Note: You may use your work from a previous question and some logic to answer this, instead of working it out from scratch.)

4) (20 pts - Counting) Consider the following set of cities: s, a_1, a_2, a_3, a_4, b_1, b_2, b_3, c_1, c_2, c_3, c_4, c_5, d_1, d_2, and t. Let there be a single one way road from s to each city in the set $\{a_i \mid 1 \le i \le 4\}$. Let there be a one way road from each city a_i to each city b_j for all $1 \le i \le 4$ and $1 \le j \le 3$. Let there be a one way road from each city b_i to each city c_j for all $1 \le i \le 3$ and $1 \le j \le 5$. Let there be a one way road from each city c_i to each city d_j for all $1 \le i \le 5$ and $1 \le j \le 2$. Finally let there be two more one way roads, one from d_1 to t and one from d_2 to t. **<u>(Put a box around your answers.)</u>**

a) (3 pts) How many different routes can someone take from s to t?

b) (2 pts) How many one way roads are there in all?

c) (3 pts) How many paths from s to t go through b_1?

d) (3 pts) How many paths from s to t go through both b_3 and d_1?

e) (5 pts) Consider adding a road from s to b_2 and one other road from c_3 to t. With these two road additions, how many different ways are there to travel between s and t?

f) (3 pts) Assume that each road takes 5 minutes to travel. With the two added roads in part e, what is the minimum travel time from s to t?

g) (1 pt) How many different paths from s to t achieve the minimum travel time in part f?

5) (10 pts - Relations) Determine if the relation R defined below over the set A = {1, 2, 3, 4} is (a) reflexive, (b) irreflexive, (c) symmetric, (d) anti-symmetric and (e) transitive or not. **Please circle your answer choice and then provide proof of your answer.**

R = {(2, 2), (4, 1), (3, 4), (2, 3), (1, 4), (3, 3), (1, 3), (2, 4)}

Reflexive: Yes No
Proof:

Irreflexive: Yes No
Proof:

Symmetric: Yes No
Proof:

Antisymmetric: Yes No
Proof:

Transitive: Yes No
Proof:

6) (14 pts - Sets) Prove or disprove the following two statements about finite sets A, B, C and D. **Please circle whether you think the statement is true or not and then provide your proof/disproof below.**

a) If $A = B$, then $A \cup C = B \cup C$.

Statement is: true false

b) If $A \cup C = B \cup C$, then $A = B$.

Statement is: true false

7) (1 pt - For Fun) What company sponsors the Coca-Cola Orlando Eye? ___________________

<u>Spring 2017 COT 3100 Final Exam: Part B (for those who didn't get Com Serv Credit)</u>

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8) (12 pts - Logic) Use the rules of inference and laws of logic to show the following argument:

$p \to r$
$q \to r$
$\bar{r}$
$\bar{s} \to p \vee q$
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s

9) (12 pts - Functions) Find the inverse of the following function:

$f(x) = 3x^2 + 24x + 13$, with a domain of all real x, $x \leq -4$.

10) (1 pt - Just for Fun) After what former mayor of Orlando is Bill Frederick Park named?

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