2019 Fall COT 3100 Section 2 Final Exam

Last Name: ___________________, First Name : _____________________

Please show your work and put a box around your final answer for each question.

1) (8 pts) A swimming pool is the shape of a rectangular prism with a length of 12 feet, a width of 10 feet and a depth of 8 feet. The pool is full at Tuesday at 8 am but springs a leak from the bottom of the pool surface that leaks 1 cubic inch of water per second into the ground. (This means that slowly, the water level in the pool decreases.) How much lower (in inches) is the water level at Wednesday morning at 8 am as compared to Tuesday at 8 am when the pool was full? Which piece of information given in the problem is mostly irrelevant?

2) (8 pts) Let the roots of the quadratic equation $f(x) = x^2 + ax + b$ be r_1 and r_2. What is the quadratic equation with leading coefficient 1 that has roots r_1^2 and r_1^2? Please give your answer in terms of a and b only.

3) (8 pts) Consider the following premises involving Boolean variables p, q, r, and s:

$$p \to (q \land r)$$
$$q \to s$$
$$\bar{r}$$

Can we conclude $\bar{s}$? If so, prove this conclusion via the rules of inference. If not, show a single truth setting such that the three given premises are true but the conclusion $\bar{s}$ is false.

4) (8 pts) Bytelandia has coins with the following denominations: 1 cent, 8 cents, 32 cents and 44 cents. Bytesar has coins that add up to exactly 1239 cents. What is the minimum number of 1 cent coins he could have? Give a set of coins that adds up to 1239 cents which achieves this minimum number of 1 cent coins and prove that it's impossible for another combination to have fewer 1 cent coins.

5) (8 pts) Prove or disprove for finite sets A, B and C: if $A \cap B = C \cap B$, then $A = C$.

6) (8 pts) Prove or disprove for finite sets A, B and C: if $A \subseteq C$, then $A \cap B \subseteq C \cap B$.

7) (15 pts) Find all integer solutions for x and y to the equation $297x + 234y = 36$.

8) (10 pts) The student government at a particular school has 1 president, 2 vice presidents, and a council of 5 students. If the school has 100 students, how many possible combinations of students can be elected to the student government? Two governments are different if they either have different presidents, or if one of the vice presidents between the governments differs, or if one of the council members between the two governments differs. (For example, let ten of the students be A, B, C, D, E, F, G, H, I and J. The government of president = A, VPs = C, E, Council = F, G, H, I, J is different than president = A, VPs = C, E, and Council = D, F, G, H and I. But, the government of president = A, VPs = E, C, Council = J, I, G, H, and F is the same as the first government listed.)

9) (12 pts) Joanna has 12 chocolates that she would like to eat within a 40 day time span. To make sure that she doesn’t eat too many all at once, she is restricting herself to eat no more than 1 a day, and also to never eat chocolates on consecutive days. An example of a valid schedule of eating the chocolates is to have a chocolate on the following days 3, 6, 8, 10, 15, 17, 19, 22, 29, 33, 36, and 40. How many valid schedules of eating the chocolates are possible, given Joanna’s restrictions? (Two schedules are different if one schedule has her eating a chocolate on a day that the other schedule doesn’t.)

10) (8 pts) 1% of the population has a genetic disease. Of the people with the disease, 96% of them have long ears while of the people without the disease, only 6% have long ears. A person with long ears is chosen at random. What is the probability she has the genetic disease? Express your answer as a fraction in lowest terms.

11) (8 pts) A bag of skittles has 10 green skittles and 20 red skittles. Johnny randomly grabs 8 skittles from the bag without looking. What is the probability that he grabs exactly 3 green skittles and 5 red skittles?

12) (10 pts) Let $f(x) = \frac{a}{x+a}$, where a is a positive constant with a domain of all real x except x= -a. For this domain, determine $f^{-1}(x)$. What are the domain and range of $f^{-1}(x)$?

13) (3 pts) What is the remainder when 3^{10003} is divided by 10?

14) (10 pts) Define the following relation R over the set of positive integers:

R = { (a, b) | gcd(a, b) > 1 }

With proof, determine if R is (a) reflexive, (b) irreflexive, (c) symmetric, (d) anti-symmetric and (e) transitive.

15) (1 pt) In 2003, December 5^{th} was anointed by Ninja Burger as "International Ninja Day", to celebrate how quickly they deliver their food. What is one food item that Ninja Burger delivers quickly?

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Scratch Page - Please carefully mark any work on this page you would like graded.