Fall 2016 COT 3100 Exam *#3* - Counting, Probability

𝑽𝑬𝑹𝑺𝑰𝑶𝑵 𝑨

Please double check that you bubbled your PID and exam version on your scantron. If you do not, you will get a 𝐙𝐄𝐑𝐎 on this exam.

1) Marie wants to buy an outfit for school. She has a choice amongst 10 blouses and 7 skirts. A valid outfit consists of one blouse and one skirt. How many different valid outfits could she buy?

a) 1　　　　　b) 17　　　　c) 70　　　　d) $\binom{10}{7}$　　　　e) None of the Above

2) UCF Mobile is a new cell phone company. They have been allocated a set of phone numbers that they can assign to their customers. (Recall that a full phone number has a 3 digit area code followed by a 7 digit number.) All of their numbers must be in the 321 or 407 area codes. The 7 digit numbers that correspond to the 321 area code must start with the digits 68 and can not have the digit 0. All of their numbers that correspond to the 407 area code must start with the digits 758 and have the same digit as their fourth and seventh digits. How many phone numbers can UCF Mobile assign?

a) $_9P_5 + {}_{10}P_3$　　b) $_9P_5 + 10^3$　　c) $10^5 + 10^3$　　d) $9^5 + 10^4$　　e) None of the Above

3) The UCF Lottery (now used to fund extra TAs for courses =)) allows participants to choose 5 numbers out of the integers 1 through 30, inclusive. Any ticket where 3 of the 5 numbers match the winning 5 numbers (all distinct, in between 1 and 30, inclusive), wins a prize of $10. How many different ticket choices match precisely 3 of the winning numbers, earning $10?

a) $\binom{30}{3}$　　　　b) $\binom{30}{5}$　　　　c)$10\binom{25}{2}$　　　d) $\binom{10}{2}\binom{20}{3}$　　e) None of the Above

4) Seema's class has 10 students in it. Each student must pick a favorite animal out of 7 possible choices and each student is free to choose any of the 7 animals. How many combinations of choices can Seema's class make? (Note: the students in the class are distinguishable and we count a combination as being different from another one if at least one particular student has a different choice of animal in the two combinations.)

a) 7^{10}　　　　　b) 10^7　　　　c) $\binom{10}{7}$　　　　d) $\binom{17}{7}$　　　　e) None of the Above

5) How many solutions does the equation

$$a + b + c + d + e = 20$$

have given that a, b, c, d, and e are each **_positive_** integers?

a) $\binom{19}{4}$ b) $\binom{20}{4}$ c) $\binom{20}{5}$ d) $\binom{24}{4}$ e) None of the Above

6) An ant is walking on the Cartesian plane, starting at the coordinate (0, 0), always moving one unit in the positive x-axis or one unit in the positive y-axis in sequence. The ant wants to end up at (12, 8) but wants to avoid the poison set up at (3, 6) as well as the anteater who is staking out the location (7, 2). In how many ways can the ant walk to her destination?

a) $\binom{20}{8}$ b) $\binom{20}{8} - \binom{9}{3}\binom{11}{9} - \binom{9}{2}\binom{11}{6}$ c) $\binom{20}{8} - \binom{9}{3}\binom{11}{5} - \binom{9}{2}\binom{11}{6}$

d) $\binom{20}{8} - \binom{9}{3}\binom{11}{9} - \binom{9}{2}\binom{11}{6} + \binom{9}{3}\binom{8}{4}\binom{11}{5}$ e) None of the Above

7) How many solutions does the inequality

$$a + b + c + d + e + f < 100$$

have given that a, b, c, d, e and f are each non-negative integers?

a) $\binom{105}{5}$ b) $\binom{105}{6}$ c) $\binom{106}{5}$ d) $\binom{106}{6}$ e) None of the Above

8) We call a number "ordered" if each of its digits are in numerical order. For example, 12234 and 25667 are both ordered, but 2344768 is not, since the 6 comes after the 7. How many ordered numbers are there with 12 digits?

a) $\binom{12}{8}$ b) $\binom{12}{9}$ c) $\binom{21}{8}$ d) $\binom{21}{9}$ e) None of the Above

9) When rolling a fair pair of standard six-sided dice, what is the probability of obtaining a sum of 5?

a) $\frac{1}{36}$ b) $\frac{1}{12}$ c) $\frac{1}{11}$ d) $\frac{1}{9}$ e) None of the Above

10) Johnny comes to class on time 40% of the time. Given that he comes to class on time on an exam day, he has an 80% chance of getting an A on the exam. If he comes late to the class on an exam day, he only has a 30% chance of getting an A on the exam. Johnny earned an A on his statics exam. What is the probability he came to class on time to take his statics exam?

a) $\frac{8}{25}$　　　　b) $\frac{1}{2}$　　　　c) $\frac{16}{25}$　　　　d) $\frac{8}{11}$　　　　e) None of the Above

11) Sharonda makes 80% of her penalty kicks. In a single game she had the opportunity to take 4 penalty kicks. What is the probability she made precisely two of them?

a) $(.8)^2(.2)^2$　　b) $2(.8)^2(.2)^2$　　c) $4(.8)^2(.2)^2$　　d) $6(.8)^2(.2)^2$　　e) None of the Above

12) A box contains 4 shiny pennies and 4 dull pennies. You pull each coin out of the box, one by one in random order. What is the probability that you pull at least one shiny penny *after* the fifth penny selected?

a) $\frac{1}{2}$　　　　b) $\frac{3}{4}$　　　　c) $\frac{6}{7}$　　　　d) $\frac{7}{8}$　　　　e) None of the Above

13) One six-sided die has the following labels on each side: 2, 3, 3, 4, 5 and 6. A second six-sided die has the following labels on each side: 1, 2, 3, 4, 4, and 5. What is the probability that the sum obtained by adding the labels that show on the top of both dice when thrown (assuming that each of the six sides on both dice are equally likely to land on the top) is 7?

a) $\frac{1}{6}$　　　　b) $\frac{7}{36}$　　　　c) $\frac{2}{9}$　　　　d) $\frac{5}{18}$　　　　e) None of the Above

14) Let X be the Discrete Random Variable defined below. What is E(X)?

$$X = \begin{array}{l} 3, \text{ with probability } \frac{1}{3} \\ 5, \text{ with probability } \frac{2}{5} \\ 7, \text{ with probability } p \end{array}$$

Note: X can only equal 3, 5 or 7 and the value of p can be calculated with the given information.

a) 4　　　　b) $4\frac{13}{15}$　　　　c) 5　　　　d) $5\frac{1}{3}$　　　　e) None of the Above

15) Which of the following expressions is equal to Var(X) for all discrete random variables X?

a) $E(X^2) - E(X)$ b) $E(X^2) - [E(X)]^2$ c) $E(X^2)$ d) $E(X^2) + E(X)$

e) None of the Above

16) By what initials is Bayerische Motoren Werke AG better known?

a) BMW   b) AUDI   c) HONDA   d) RAM   e) None of the Above